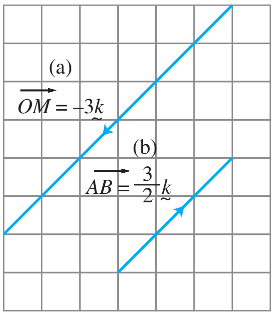
Fully-Worked Solutions

FORM 4

CHAPTER 8 Vectors

Self Test 1

1



2 $\underset{\sim}{m} = 2\underset{\sim}{a} + 3\underset{\sim}{b}$

$\underset{\sim}{n} = -4\underset{\sim}{a} + \underset{\sim}{b}$

$c\underset{\sim}{m} + d\underset{\sim}{n} = c(2\underset{\sim}{a} + 3\underset{\sim}{b}) + d(-4\underset{\sim}{a} + \underset{\sim}{b})$

$= (2c - 4d)\underset{\sim}{a} + (3c + d)\underset{\sim}{b}$

Compare with $-2\underset{\sim}{a} + 11\underset{\sim}{b}$,

$2c - 4d = -2$

$c - 2d = -1$

$c = 2d - 1$ ...... ①

From ①,

$3c + d = 11$

$3(2d - 1) + d = 11$

$6d - 3 + d = 11$

$7d = 14$

$d = 2$

$c = 2(2) - 1 = 3$

3 (a) $\overrightarrow{AB} = -4\underset{\sim}{a}, \overrightarrow{CD} = -\frac{5}{2}\underset{\sim}{a}$

$-4\underset{\sim}{a} = \frac{8}{5}\left(-\frac{5}{2}\underset{\sim}{a}\right) \Rightarrow \overrightarrow{AB} = \frac{8}{5}\overrightarrow{CD}$

Thus, $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel.

(b) $\overrightarrow{RS} = \frac{2}{3}k\underset{\sim}{p} - 6\underset{\sim}{q}, = \overrightarrow{HT} = -\frac{1}{5}\underset{\sim}{p} + 2k\underset{\sim}{q}$

$\overrightarrow{RS} = m\overrightarrow{HT}$

$\frac{2}{3}k\underset{\sim}{p} - 6\underset{\sim}{q}, = m\left(-\frac{1}{5}\underset{\sim}{p} + 2k\underset{\sim}{q}\right)$

$\frac{2}{3}k = -\frac{1}{5}m$

$k = -\frac{3}{10}m$ .... ①

$-6 = 2mk$

From ①,

$-6 = 2m\left(-\frac{3}{10}m\right)$

$m^2 = 10$

$m = \sqrt{10}$

$\therefore k = -\frac{3}{10} \times m = -\frac{3}{10}\sqrt{10}$

Self Test 2

1 $\underset{\sim}{p} = 5\underset{\sim}{a} + (x + y)\underset{\sim}{b}, \underset{\sim}{q} = 16x\underset{\sim}{a} - \underset{\sim}{b}$

(a) $2\underset{\sim}{p} - \underset{\sim}{q} = 2[5\underset{\sim}{a} + (x + y)\underset{\sim}{b}] - (16x\underset{\sim}{a} - \underset{\sim}{b})$

$= 10\underset{\sim}{a} + 2(x + y)\underset{\sim}{b} - 16x\underset{\sim}{a} + \underset{\sim}{b}$

$8\underset{\sim}{a} + \frac{4}{3}\underset{\sim}{b} = (10 - 16x)\underset{\sim}{a} + (2x + 2y + 1)\underset{\sim}{b}$

$10 - 16x = 8$

$16x = 2$

$x = \frac{1}{8}$

$2x + 2y + 1 = \frac{4}{3}$

$2\left(\frac{1}{8}\right) + 2y = \frac{1}{3}$

$2y = \frac{1}{12}$

$y = \frac{1}{24}$

(b) $\underset{\sim}{p} = 5\underset{\sim}{a} + \frac{1}{6}\underset{\sim}{b}, \underset{\sim}{q} = 2\underset{\sim}{a} - \underset{\sim}{b}$

(i) $2\underset{\sim}{p} + 3\underset{\sim}{q} = 2\left(5\underset{\sim}{a} + \frac{1}{6}\underset{\sim}{b}\right) + 3(2\underset{\sim}{a} - \underset{\sim}{b})$

$= 10\underset{\sim}{a} + \frac{1}{3}\underset{\sim}{b} + 6\underset{\sim}{a} - 3\underset{\sim}{b}$

$= 16\underset{\sim}{a} - \frac{8}{3}\underset{\sim}{b}$

(ii) $3\underset{\sim}{p} - 5\underset{\sim}{q} = 3\left(5\underset{\sim}{a} + \frac{1}{6}\underset{\sim}{b}\right) - 5(2\underset{\sim}{a} - \underset{\sim}{b})$

$= 15\underset{\sim}{a} - 10\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b} + 5\underset{\sim}{b}$

$= 5\underset{\sim}{a} + \frac{11}{2}\underset{\sim}{b}$

2 (a) $\overrightarrow{SQ} = \overrightarrow{SP} + \overrightarrow{PQ}$

$= \underset{\sim}{b} + \underset{\sim}{a}$

(b) $\overrightarrow{TQ} = \frac{1}{3}\overrightarrow{SQ}$

$= \frac{1}{3}(\underset{\sim}{b} + \underset{\sim}{a})$

(c) $\overrightarrow{RQ} = \overrightarrow{RS} + \overrightarrow{SQ}$

$= -2\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{a}$

$= -\underset{\sim}{a} + \underset{\sim}{b}$

(d) $\overrightarrow{PT} = \overrightarrow{PQ} + \overrightarrow{QT}$

$= \underset{\sim}{a} + \left(-\frac{1}{3}\right)(\underset{\sim}{b} + \underset{\sim}{a})$

$= \frac{2}{3}\underset{\sim}{a} - \frac{1}{3}\underset{\sim}{b}$

(e) $\overrightarrow{TR} = \overrightarrow{TQ} + \overrightarrow{QR}$

$= \frac{1}{3}(\underset{\sim}{b} + \underset{\sim}{a}) + (\underset{\sim}{a} - \underset{\sim}{b})$

$= \frac{4}{3}\underset{\sim}{a} - \frac{2}{3}\underset{\sim}{b}$

$2\overrightarrow{PT} = 2\left(\frac{2}{3}\underset{\sim}{a} - \frac{1}{3}\underset{\sim}{b}\right)$

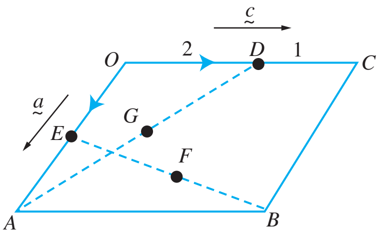
$\overrightarrow{TR} = \frac{4}{3}\underset{\sim}{a} - \frac{2}{3}\underset{\sim}{b}$

$= 2\left(\frac{2}{3}\underset{\sim}{a} - \frac{1}{3}\underset{\sim}{b}\right)$

$\overrightarrow{TR} = 2\overrightarrow{PT}$

Thus, P, T and R are collinear.

3



(b) $\overrightarrow{BE} = \overrightarrow{BA} + \overrightarrow{AE}$

$= -\underset{\sim}{c} - \frac{1}{2}\underset{\sim}{a}$

(b) $\overrightarrow{DA} = \overrightarrow{DO} + \overrightarrow{OA}$

$= -\frac{2}{3}\overrightarrow{OC} + \overrightarrow{OA}$

$= -\frac{2}{3}\underset{\sim}{c} + \underset{\sim}{a}$

(c) $\overrightarrow{GF} = \overrightarrow{GA} + \overrightarrow{AB} + \overrightarrow{BF}$

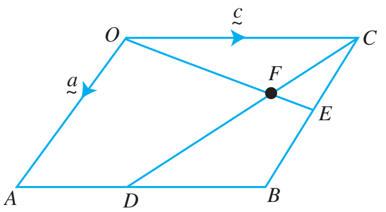
$= \frac{1}{2}\overrightarrow{DA} + \overrightarrow{AB} + \frac{1}{2}\overrightarrow{BE}$

$= \frac{1}{2}\left(-\frac{2}{3}\underset{\sim}{c} + \underset{\sim}{a}\right) + \underset{\sim}{c} + \frac{1}{2}\left(-\underset{\sim}{c} - \frac{1}{2}\underset{\sim}{a}\right)$

$= -\frac{1}{3}\underset{\sim}{c} + \frac{1}{2}\underset{\sim}{a} + \underset{\sim}{c} - \frac{1}{2}\underset{\sim}{c} - \frac{1}{4}\underset{\sim}{a}$

$= \frac{1}{4}\underset{\sim}{a} + \frac{1}{6}\underset{\sim}{c}$

4



(a) (i) $\overrightarrow{DC} = \overrightarrow{DB} + \overrightarrow{BC}$

$= \frac{1}{2}\overrightarrow{AB} + \overrightarrow{BC}$

$= \frac{1}{2}\underset{\sim}{c} - \underset{\sim}{a}$

(ii) $\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BE}$

$= \underset{\sim}{a} + \underset{\sim}{c} - \frac{1}{2}\underset{\sim}{a}$

$= \frac{1}{2}\underset{\sim}{a} + \underset{\sim}{c}$

(b) $\overrightarrow{DF} = h\overrightarrow{DC}$, $\overrightarrow{OF} = k\overrightarrow{OE}$

$\overrightarrow{DF} = \overrightarrow{DA} + \overrightarrow{AO} + \overrightarrow{OF}$

$= -\frac{1}{2}\underset{\sim}{c} - \underset{\sim}{a} + k\left(\frac{1}{2}\underset{\sim}{a} + \underset{\sim}{c}\right)$

$= \left(\frac{1}{2}k - 1\right)\underset{\sim}{a} + \left(k - \frac{1}{2}\right)\underset{\sim}{c}$ .... ①

$\overrightarrow{DF} = h\left(\frac{1}{2}\underset{\sim}{c} - \underset{\sim}{a}\right)$ .... ②

Compare ① with ②,

$\frac{1}{2}k - 1 = -h$

$\frac{1}{2}\left(\frac{1}{2}h + \frac{1}{2}\right) - 1 = -h$

$\frac{1}{4}h + \frac{1}{4} - 1 = -h$

$\frac{5}{4}h = \frac{3}{4}$

$h = \frac{3}{5}$

$k - \frac{1}{2} = \frac{1}{2}h$

$k = \frac{1}{2}h + \frac{1}{2}$

$= \frac{1}{2}\left(\frac{3}{5}\right) + \frac{1}{2}$

$= \frac{4}{5}$

Self Test 3

1 (a) (i) $\overrightarrow{AO} = -\overrightarrow{OA}$

$= -(-2\underset{\sim}{i} + 6\underset{\sim}{j})$

$= 2\underset{\sim}{i} - 6\underset{\sim}{j}$

(ii) $\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$

$= -\overrightarrow{OB} + \overrightarrow{OC}$

$= -(5\underset{\sim}{i} + \underset{\sim}{j}) + 2\underset{\sim}{i} + 10\underset{\sim}{j}$

$= -3\underset{\sim}{i} + 9\underset{\sim}{j}$

(b) $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$

$= 2\underset{\sim}{i} - 6\underset{\sim}{j} + 2\underset{\sim}{i} + 10\underset{\sim}{j}$

$= 4\underset{\sim}{i} + 4\underset{\sim}{j}$

Magnitude of $\overrightarrow{AC} = \sqrt{4^2 + 4^2}$

$= \sqrt{32}$

$= 4\sqrt{2}$ units

2 (a) $2\underset{\sim}{p} - \underset{\sim}{q} = 2\begin{pmatrix}5\\2\end{pmatrix} - \begin{pmatrix}-3\\-2\end{pmatrix}$

$= \begin{pmatrix}10 + 3\\4 + 2\end{pmatrix}$

$= \begin{pmatrix}13\\6\end{pmatrix}$

$|2\underset{\sim}{p} - \underset{\sim}{q}| = \sqrt{13^2 + 6^2}$

$= \sqrt{205}$ units

(b) $\begin{pmatrix}m\\n\end{pmatrix} = 2\begin{pmatrix}5\\2\end{pmatrix} + 3\begin{pmatrix}-3\\-2\end{pmatrix}$

$= \begin{pmatrix}10 - 9\\4 - 6\end{pmatrix}$

$= \begin{pmatrix}1\\-2\end{pmatrix}$

$\therefore m = 1, n = -2$

(c) $\underset{\sim}{p} - 4\underset{\sim}{q} = \begin{pmatrix}5\\2\end{pmatrix} - 4\begin{pmatrix}-3\\-2\end{pmatrix}$

$= \begin{pmatrix}5 + 12\\2 + 8\end{pmatrix}$

$= \begin{pmatrix}17\\10\end{pmatrix}$

$|\underset{\sim}{p} - 4\underset{\sim}{q}| = \sqrt{17^2 + 10^2}$

$= \sqrt{389}$ units

Unit vector $= \frac{17\underset{\sim}{i} + 10\underset{\sim}{j}}{\sqrt{389}}$

$= \frac{17}{\sqrt{389}}\underset{\sim}{i} + \frac{10}{\sqrt{389}}\underset{\sim}{j}$

3 $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$

(a) $\begin{pmatrix}4\\5\end{pmatrix} = -\begin{pmatrix}-6\\-2\end{pmatrix} + \begin{pmatrix}x\\y\end{pmatrix}$

$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}4\\5\end{pmatrix} + \begin{pmatrix}-6\\-2\end{pmatrix}$

$= \begin{pmatrix}-2\\3\end{pmatrix}$

$\therefore Q(-2, 3)$

(b) $|\overrightarrow{PQ}| = \sqrt{4^2 + 5^2}$
$= \sqrt{41}$ units

(c) $|\overrightarrow{OQ}| = \sqrt{(-2)^2 + 3^2}$
$= \sqrt{13}$ units

Unit vector $= \dfrac{-2\underset{\sim}{i} + 3\underset{\sim}{j}}{\sqrt{13}}$
$= -\dfrac{2}{\sqrt{13}}\underset{\sim}{i} + \dfrac{3}{\sqrt{13}}\underset{\sim}{j}$

(d) $\overrightarrow{PQ} = k\overrightarrow{RS}$
$\begin{pmatrix}4\\5\end{pmatrix} = k\begin{pmatrix}5\\a\end{pmatrix}$
$4 = 5k$ $\quad$ $5 = ak$
$k = \dfrac{4}{5}$ $\quad$ $5 = a\left(\dfrac{5}{4}\right)$
$a = 4$

SPM Practice

Paper 1

1 $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$
$= 6\underset{\sim}{x} - 8\underset{\sim}{y} + 2p\underset{\sim}{x} + 3\underset{\sim}{y}$
$= (6 + 2p)\underset{\sim}{x} - 5\underset{\sim}{y}$
$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$
$= -2p\underset{\sim}{x} - 3\underset{\sim}{y} + 5\underset{\sim}{x} - \underset{\sim}{y}$
$= (5 - 2p)\underset{\sim}{x} - 4\underset{\sim}{y}$
$\overrightarrow{AB} = m\overrightarrow{BC}$
$(6 + 2p)\underset{\sim}{x} - 5\underset{\sim}{y} = m[(5 - 2p)\underset{\sim}{x} - 4\underset{\sim}{y}]$
$-5 = -4m$
$m = \dfrac{5}{4}$
$6 + 2p = m(5 - 2p)$
$6 + 2p = \dfrac{5}{4}(5 - 2p)$
$24 + 8p = 25 - 10p$
$18p = 1$
$p = \dfrac{1}{18}$

2 (a) $2\overrightarrow{WX} = 3\overrightarrow{XY}$
$\overrightarrow{XY} = \dfrac{2}{3}\overrightarrow{WX}$
$= \dfrac{2}{3}\underset{\sim}{x}$

(b) $\overrightarrow{VB} = \dfrac{1}{3}\overrightarrow{BY}$
$\overrightarrow{VY} = \overrightarrow{VW} + \overrightarrow{WY}$
$= -\underset{\sim}{y} + \underset{\sim}{x} + \dfrac{2}{3}\underset{\sim}{x}$
$= -\underset{\sim}{y} + \dfrac{5}{3}\underset{\sim}{x}$
$\overrightarrow{VB} = \dfrac{1}{4}\overrightarrow{VY}$
$= \dfrac{1}{4}\left(-\underset{\sim}{y} + \dfrac{5}{3}\underset{\sim}{x}\right)$
$= \dfrac{5}{12}\underset{\sim}{x} - \dfrac{1}{4}\underset{\sim}{y}$

3 $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$
$= -\begin{pmatrix}2\\1\end{pmatrix} + \begin{pmatrix}1\\3\end{pmatrix}$
$= \begin{pmatrix}-1\\2\end{pmatrix}$

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$
$= -\begin{pmatrix}2\\1\end{pmatrix} + \begin{pmatrix}5\\-k\end{pmatrix}$
$= \begin{pmatrix}3\\-1-k\end{pmatrix}$

(a) A, B and C collinear $\Rightarrow \overrightarrow{AB} = m\overrightarrow{AC}$
$\begin{pmatrix}-1\\2\end{pmatrix} = m\begin{pmatrix}3\\-1-k\end{pmatrix}$
$-1 = 3m$ $\quad$ $2 = m(-1 - k)$
$m = -\dfrac{1}{3}$ $\quad$ $2 = \left(-\dfrac{1}{3}\right)(-1 - k)$
$6 = 1 + k$
$k = 5$

(b) $\dfrac{\overrightarrow{AB}}{\overrightarrow{AC}} = m = \dfrac{1}{3}$ (in opposite direction)

4 (a) Magnitude of $\begin{pmatrix}-5\\-12\end{pmatrix} = \sqrt{(-5)^2 + (-12)^2}$
$= 13$ units

The vector with the same direction as vector $\begin{pmatrix}4\\3\end{pmatrix}$ and a magnitude of 13
$= 13\begin{pmatrix}4\\3\end{pmatrix}$
$= 52\underset{\sim}{i} + 39\underset{\sim}{j}$

(b) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$
$= -\begin{pmatrix}-6\\-3\end{pmatrix} + \begin{pmatrix}3\\9\end{pmatrix}$
$= \begin{pmatrix}9\\12\end{pmatrix}$

Magnitude of $AB = \sqrt{9^2 + 12^2} = 15$ units

Unit vector $= \dfrac{9}{15}\underset{\sim}{i} + \dfrac{12}{15}\underset{\sim}{j}$

5 $\overrightarrow{OA} = \begin{pmatrix}6\\-2\end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix}4\\12\end{pmatrix}$

(a) Point $C = \left(\dfrac{6 + 4}{2}, \dfrac{-2 + 12}{2}\right)$
$= (5, 5)$
$\therefore \overrightarrow{OC} = 5\underset{\sim}{i} + 5\underset{\sim}{j}$

(b) $\overrightarrow{CA} = 3\overrightarrow{CB}$
$\overrightarrow{CO} + \overrightarrow{OA} = 3(\overrightarrow{CO} + \overrightarrow{OB})$
$\overrightarrow{OA} - 3\overrightarrow{OB} = 2\overrightarrow{CO}$
$\begin{pmatrix}6\\-2\end{pmatrix} - 3\begin{pmatrix}4\\12\end{pmatrix} = 2\begin{pmatrix}-x\\-y\end{pmatrix}$
$-2\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}6-12\\-2-36\end{pmatrix}$
$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}3\\19\end{pmatrix}$
$\therefore \overrightarrow{OC} = 3\underset{\sim}{i} + 19\underset{\sim}{j}$

(c) $\overrightarrow{OC} = \overrightarrow{AB}$
$= \overrightarrow{AO} + \overrightarrow{OB}$
$= -\begin{pmatrix}6\\-2\end{pmatrix} + \begin{pmatrix}4\\12\end{pmatrix}$
$= \begin{pmatrix}-6+4\\2+12\end{pmatrix}$
$= \begin{pmatrix}-2\\14\end{pmatrix}$
$\therefore \overrightarrow{OC} = -2\underset{\sim}{i} + 14\underset{\sim}{j}$

6 (a) $\overrightarrow{OM} = \begin{pmatrix}2\\-3\end{pmatrix}$, $\overrightarrow{ON} = \begin{pmatrix}1\\-1\end{pmatrix}$, $\overrightarrow{OP} = \begin{pmatrix}2p\\4\end{pmatrix}$
$\overrightarrow{MN} = \overrightarrow{MO} + \overrightarrow{ON}$
$= \begin{pmatrix}-2\\3\end{pmatrix} + \begin{pmatrix}1\\-1\end{pmatrix}$
$= \begin{pmatrix}-1\\2\end{pmatrix}$
$\overrightarrow{NP} = \overrightarrow{NO} + \overrightarrow{OP}$
$= \begin{pmatrix}-1\\1\end{pmatrix} + \begin{pmatrix}2p\\4\end{pmatrix}$
$= \begin{pmatrix}2p-1\\5\end{pmatrix}$

M, N and P are collinear, thus

$\overrightarrow{MN} = k\overrightarrow{NP}$

$\begin{pmatrix} -1 \\ 2 \end{pmatrix} = k\begin{pmatrix} 2p-1 \\ 5 \end{pmatrix}$

$2 = 5k \qquad -1 = k(2p-1)$

$k = \frac{2}{5} \qquad -1 = \frac{2}{5}(2p-1)$

$-5 = 4p - 2$

$p = -\frac{3}{4}$

(b) $\overrightarrow{NP} = \begin{pmatrix} 2p-1 \\ 5 \end{pmatrix}, \overrightarrow{MN} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$|\overrightarrow{NP}| = \sqrt{(2p-1)^2 + 5^2}$

$|\overrightarrow{MN}| = \sqrt{(-1)^2 + 2^2} = \sqrt{5}$

$|\overrightarrow{NP}| = \sqrt{5}|\overrightarrow{MN}|$

$= \sqrt{5} \times \sqrt{5}$

$= 5$

$\sqrt{(2p-1)^2 + 5^2} = 5$

$(2p-1)^2 + 5^2 = 25$

$(2p-1)^2 = 0$

$p = \frac{1}{2}$

7 Midpoint of $BC = \left(\frac{2+2}{2}, \frac{10+1}{2}\right)$

$= \left(2, \frac{11}{2}\right)$

Midpoint of $AD = \left(\frac{x+6}{2}, \frac{y+4}{2}\right) = \left(2, \frac{11}{2}\right)$

$\frac{x+6}{2} = 2 \qquad \frac{y+4}{2} = \frac{11}{2}$

$x = -2 \qquad y = 7$

$\therefore A(-2, 7)$

(a) $\overrightarrow{CD} = \overrightarrow{CO} + \overrightarrow{OD}$

$= \begin{pmatrix} -2 \\ -1 \end{pmatrix} + \begin{pmatrix} 6 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$= 4\underset{\sim}{i} + 3\underset{\sim}{j}$

(b) $\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD}$

$= \begin{pmatrix} 2 \\ -7 \end{pmatrix} + \begin{pmatrix} 6 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 8 \\ -3 \end{pmatrix}$

$= 8\underset{\sim}{i} - 3\underset{\sim}{j}$

8 (a) Unit vector: Magnitude = 1

$\sqrt{0.4^2 + x^2} = 1$

$0.16 + x^2 = 1$

$x^2 = 0.84$

$x = 0.92$

(b) (i) $3\underset{\sim}{a} + \underset{\sim}{b} - 2\underset{\sim}{c} = 3\begin{pmatrix} -3 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} - 2\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} -9 \\ 12 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 6 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} -13 \\ 17 \end{pmatrix}$

$|3\underset{\sim}{a} + \underset{\sim}{b} - 2\underset{\sim}{c}| = \sqrt{(-13)^2 + 17^2}$

$= \sqrt{458}$ units

(ii) $\underset{\sim}{c} - \underset{\sim}{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ -6 \end{pmatrix}$

$|\underset{\sim}{c} - \underset{\sim}{a}| = \sqrt{36 + 36}$

$= \sqrt{2 \times 36}$

$= 6\sqrt{2}$

Unit vector $= \frac{6\underset{\sim}{i} - 6\underset{\sim}{j}}{6\sqrt{2}} = \frac{1}{\sqrt{2}}\underset{\sim}{i} - \frac{1}{\sqrt{2}}\underset{\sim}{j}$

Paper 2

1 (a) $\overrightarrow{DB} = \overrightarrow{DC} + \overrightarrow{CB}$

$= 7\underset{\sim}{a} - \underset{\sim}{b}$

$\overrightarrow{DA} = \overrightarrow{DC} + \overrightarrow{CB} + \overrightarrow{BA}$

$= 7\underset{\sim}{a} - \underset{\sim}{b} - 4\underset{\sim}{a}$

$= 3\underset{\sim}{a} - \underset{\sim}{b}$

(b) (i) $\overrightarrow{AX} = k\overrightarrow{AC}$

$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$

$= -\overrightarrow{DA} + \overrightarrow{DC}$

$= -3\underset{\sim}{a} + \underset{\sim}{b} + 7\underset{\sim}{a}$

$= 4\underset{\sim}{a} + \underset{\sim}{b}$

$\therefore \overrightarrow{AX} = k(4\underset{\sim}{a} + \underset{\sim}{b})$

$= 4k\underset{\sim}{a} + k\underset{\sim}{b}$

(ii) $\overrightarrow{DX} = \overrightarrow{DA} + \overrightarrow{AX}$

$= 3\underset{\sim}{a} - \underset{\sim}{b} + 4k\underset{\sim}{a} + k\underset{\sim}{b}$

$= (3 + 4k)\underset{\sim}{a} + (k-1)\underset{\sim}{b}$

(c) $\overrightarrow{DX} = m\overrightarrow{DB}$

$= m(7\underset{\sim}{a} - \underset{\sim}{b})$

$= 7m\underset{\sim}{a} - m\underset{\sim}{b}$

$\overrightarrow{DX} = (3 + 4k)\underset{\sim}{a} + (k-1)\underset{\sim}{b}$

$7m = 3 + 4k$ .... ①

$-m = k - 1$

$m = -k + 1$ .... ②

Substitute ② into ①,

$7(-k + 1) = 3 + 4k$

$-7k + 7 = 3 + 4k$

$11k = 4$

$k = \frac{4}{11}$

$m = -\frac{4}{11} + 1$

$m = \frac{7}{11}$

(i) $\overrightarrow{DX} = m\overrightarrow{DB}$

$= \frac{7}{11}(7\underset{\sim}{a} - \underset{\sim}{b})$

$= \frac{49}{11}\underset{\sim}{a} - \frac{7}{11}\underset{\sim}{b}$

$|\overrightarrow{DX}| = \sqrt{\left(\frac{49}{11}\right)^2 + \left(-\frac{7}{11}\right)^2}$

$= 4.5$ units

(ii) $\overrightarrow{AC} = 4\underset{\sim}{a} + \underset{\sim}{b}$

$|\overrightarrow{AC}| = \sqrt{4^2 + 1^2}$

$= \sqrt{17}$ units

Unit vector of $\overrightarrow{AC} = \frac{4\underset{\sim}{a} + \underset{\sim}{b}}{\sqrt{17}} = \frac{4}{\sqrt{17}}\underset{\sim}{a} + \frac{1}{\sqrt{17}}\underset{\sim}{b}$

2 (a) (i) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= \begin{pmatrix} -6 \\ 3 \end{pmatrix} + \begin{pmatrix} 15 \\ 9 \end{pmatrix}$

$= \begin{pmatrix} 9 \\ 12 \end{pmatrix}$

$|\overrightarrow{AB}| = \sqrt{9^2 + 12^2}$
$= \sqrt{225}$
$= 15$ units

Unit vector parallel to $AB = \dfrac{9\underset{\sim}{i} + 12\underset{\sim}{j}}{15} = \dfrac{3}{5}\underset{\sim}{i} + \dfrac{4}{5}\underset{\sim}{j}$

(ii) $\overrightarrow{AC} = \dfrac{2}{3}\overrightarrow{AB}$
$= \dfrac{2}{3}\begin{pmatrix} 9 \\ 12 \end{pmatrix}$
$= \begin{pmatrix} 6 \\ 8 \end{pmatrix}$

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$
$\begin{pmatrix} 6 \\ 8 \end{pmatrix} = \begin{pmatrix} -6 \\ 3 \end{pmatrix} + \overrightarrow{OC}$
$\overrightarrow{OC} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} - \begin{pmatrix} -6 \\ 3 \end{pmatrix}$
$= \begin{pmatrix} 12 \\ 5 \end{pmatrix}$

$\therefore \overrightarrow{OC} = 12\underset{\sim}{i} + 5\underset{\sim}{j}$

(b) (i) $\underset{\sim}{u} = m\underset{\sim}{v}$
$(1 + k)\underset{\sim}{i} - 2\underset{\sim}{j} = m(6\underset{\sim}{i} - k\underset{\sim}{j})$
$1 + k = 6m$ .... ①
$-2 = -mk$
$m = \dfrac{2}{k}$ .... ②

Substitute ② into ①,
$1 + k = 6\left(\dfrac{2}{k}\right)$
$k + k^2 = 12$
$k^2 + k - 12 = 0$
$(k + 4)(k - 3) = 0$
$k = -4, k = 3$

(ii) $k > 0, \therefore k = 3$
$3\underset{\sim}{u} - \underset{\sim}{v} = 3\begin{pmatrix} 1 + 3 \\ -2 \end{pmatrix} - \begin{pmatrix} 6 \\ -3 \end{pmatrix}$
$= \begin{pmatrix} 12 \\ -6 \end{pmatrix} - \begin{pmatrix} 6 \\ -3 \end{pmatrix}$
$= \begin{pmatrix} 6 \\ -3 \end{pmatrix}$

$|3\underset{\sim}{u} - \underset{\sim}{v}| = \sqrt{6^2 + (-3)^2}$
$= \sqrt{45}$
$= 3\sqrt{5}$ units

(c) (i) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$
$= \begin{pmatrix} 2 \\ -3 \end{pmatrix} + \begin{pmatrix} -6 \\ 8 \end{pmatrix}$
$= \begin{pmatrix} -4 \\ 5 \end{pmatrix}$
$= -4\underset{\sim}{i} + 5\underset{\sim}{j}$

(ii) $\overrightarrow{DC} = \overrightarrow{DO} + \overrightarrow{OC}$
$= \begin{pmatrix} -7 \\ -6 \end{pmatrix} + \begin{pmatrix} 3 \\ 11 \end{pmatrix}$
$= \begin{pmatrix} -4 \\ 5 \end{pmatrix}$
$= -4\underset{\sim}{i} + 5\underset{\sim}{j}$

Thus, vectors $\overrightarrow{AB}$ and $\overrightarrow{DC}$ are parallel vectors with the same magnitude.