FORM 4

CHAPTER 7 Coordinate Geometry

Self Test 1

1 (a) $PA = PB$

$P = \left(\frac{6+4}{2}, \frac{1+(-1)}{2}\right)$

$= (5, 0)$

(b) $\frac{PA}{PB} = \frac{1}{3}$

$P = \left(\frac{4+3(6)}{4}, \frac{-1+3}{4}\right)$

$= \left(\frac{11}{2}, \frac{1}{2}\right)$

2 $R = \left(\frac{3p+5r}{6}, \frac{-q-5q}{6}\right)$

$(2p, 2r) = \left(\frac{3p+5r}{6}, \frac{-q-5q}{6}\right)$

$2r = -\frac{6q}{6}$

$r = -\frac{1}{2}q$ .... ①

$\frac{3p+5r}{6} = 2p$

$3p + 5r = 12p$

$5r = 9p$

From ①,

$5\left(-\frac{1}{2}q\right) = 9p$

$p = -\frac{5}{18}q$

3 $5x + y = 17$

$y = 17 - 5x$

(a) $5x^2 + (17-5x)^2 = 49$

$5x^2 + 289 - 170x + 25x^2 = 49$

$30x^2 - 170x + 240 = 0$

$3x^2 - 17x + 24 = 0$

$(3x-8)(x-3) = 0$

$x = \frac{8}{3}, x = 3$

$x = \frac{8}{3}, y = 17 - 5\left(\frac{8}{3}\right) = \frac{11}{3}$

$x = 3, y = 17 - 5(3) = 2$

Thus, $A\left(\frac{8}{3}, \frac{11}{3}\right)$ and $B(3, 2)$

(b) Midpoint of $AB = \left(\frac{\frac{8}{3}+3}{2}, \frac{\frac{11}{3}+2}{2}\right)$

$= \left(\frac{17}{6}, \frac{17}{6}\right)$

(c) $P = \left(\frac{\frac{8}{3}(4)+3}{5}, \frac{\frac{11}{3}(4)+2}{5}\right)$

$= \left(\frac{41}{15}, \frac{10}{3}\right)$

Self Test 2

1 (a) Midpoint of $AB = \left(\frac{2+(-6)}{2}, \frac{3+(-4)}{2}\right)$

$= \left(-2, -\frac{1}{2}\right)$

(b) $m_{AB} = \frac{-4-3}{-6-2} = \frac{7}{8}$

$m_{\perp AB} = -\frac{8}{7}$

$\frac{y+\frac{1}{2}}{x+2} = -\frac{8}{7}$

$7y + \frac{7}{2} = -8x - 16$

$7y = -8x - \frac{39}{2}$

$y = -\frac{7}{8}x - \frac{39}{14}$

2 $TK: 4y - kx - 18 = 0$

$y = \frac{kx}{4} + \frac{18}{4}$

$y = \frac{k}{4}x + \frac{9}{2}$

$EF: \frac{y}{3} - \frac{x}{4} = 2$

$4y - 3x = 24$

$y = \frac{3}{4}x + 6$

(a) Parallel: $\frac{k}{4} = \frac{3}{4}$

$k = 3$

(b) Perpendicular: $\frac{k}{4} \times \frac{3}{4} = -1$

$\frac{k}{4} = -\frac{4}{3}$

$k = -\frac{16}{3}$

3 $3x - 2y = 5$

$2y = 3x - 5$

$y = \frac{3}{2}x - \frac{5}{2} \Rightarrow m = \frac{3}{2}$

Parallel: $\frac{y+3}{x-1} = \frac{3}{2}$

$2y + 6 = 3x - 3$

$2y = 3x - 9$

$y = \frac{3}{2}x - \frac{9}{2}$

4 $4y - 5x + 20 = 0$

(a) Point $A = (0, y)$

$4y - 5(0) + 20 = 0$

$4y = -20$

$y = -5$

$\therefore A(0, -5)$

Point $B = (x, 0)$

$4(0) - 5x + 20 = 0$

$-5x = -20$

$x = 4$

$\therefore B(4, 0)$

Point $M = \left(\frac{2(4)+0}{3}, \frac{2(0)+(-5)}{3}\right)$

$= \left(\frac{8}{3}, -\frac{5}{3}\right)$

(b) $4y = 5x - 20$

$y = \frac{5}{4}x - 5$

$m_\perp = -\frac{4}{5}$

$\frac{y + \frac{5}{3}}{x - \frac{8}{3}} = -\frac{4}{5}$

$5y + \frac{25}{3} = -4x + \frac{32}{3}$

$5y = -4x + \frac{7}{3}$

$y = -\frac{4}{5}x + \frac{7}{15}$

Self Test 3

1 (a) Area of $\triangle = \frac{1}{2}\begin{vmatrix} -5 & 2 & 4 & -5 \\ -2 & -1 & 1 & -2 \end{vmatrix}$

$= \frac{1}{2}|(5+2-8)-(-4-4-5)|$

$= \frac{1}{2}|-1-(-13)|$

$= 6 \text{ units}^2$

(b) Area of $\triangle = \frac{1}{2}\begin{vmatrix} -8 & -8 & -5 & -8 \\ 5 & 10 & -1 & 5 \end{vmatrix}$

$= \frac{1}{2}|(-80+8-25)-(-40-50+8)|$

$= \frac{1}{2}|-15|$

$= \frac{15}{2} \text{ units}^2$

2 (a) Area of $ABCD = \frac{1}{2}\begin{vmatrix} -3 & 1 & 12 & 5 & -3 \\ 8 & -3 & 2 & 10 & 8 \end{vmatrix}$

$= \frac{1}{2}|(9+2+120+40)-(8-36+10-30)|$

$= \frac{1}{2}|219|$

$= 109.5 \text{ units}^2$

(b) Area of $\triangle OCD = \frac{1}{2}\begin{vmatrix} 0 & 12 & 5 & 0 \\ 0 & 2 & 10 & 0 \end{vmatrix}$

$= \frac{1}{2}|(120)-(10)|$

$= 55 \text{ units}^2$

(c) Given $E = (x, 0)$

Area of $\triangle OED = 16$

$\frac{1}{2}\begin{vmatrix} 0 & x & 5 & 0 \\ 0 & 0 & 10 & 0 \end{vmatrix} = 16$

$|10x| = 32$

$10x = \pm 32$

$x = -\frac{16}{5}$ or $x = \frac{16}{5}$

$\therefore E = \left(-\frac{16}{5}, 0\right)$ or $E\left(\frac{16}{5}, 0\right)$

3 Area = 0 (Collinear)

$\frac{1}{2}\begin{vmatrix} -2 & 3 & -3 & -2 \\ k & 1 & -1 & k \end{vmatrix} = 0$

$|(-2-3-3k)-(3k-3+2)| = 0$

$|-5-3k-3k+1| = 0$

$|-6k-4| = 0$

$-6k-4 = 0$

$k = -\frac{2}{3}$

4 (a) RS is parallel to TU

$m_{RS} = m_{TU}$

$\frac{3-1}{-3-1} = \frac{-1-y}{0-(-5)}$

$-\frac{2}{4} = \frac{-1-y}{5}$

$10 = 4 + 4y$

$y = \frac{3}{2}$

(b) Area of trapezium $= \frac{1}{2}\begin{vmatrix} -3 & 1 & 0 & -5 & -3 \\ 3 & 1 & -1 & \frac{3}{2} & 3 \end{vmatrix}$

$= \frac{1}{2}\left|\left(-3-1-15\right)-\left(3+5-\frac{9}{2}\right)\right|$

$= \frac{1}{2}\left|-\frac{45}{2}\right|$

$= \frac{45}{4} \text{ units}^2$

Self Test 4

1 (a) Distance = 5

$\sqrt{(x+4)^2+(y-2)^2} = 5$

$(x+4)^2+(y-2)^2 = 25$

$x^2+8x+16+y^2-4y+4 = 25$

$x^2+y^2+8x-4y-5 = 0$

(b) $\frac{PA}{PB} = \frac{2}{3}$

$3PA = 2PB$

$3\sqrt{(x+4)^2+(y-2)^2} = 2\sqrt{(x-0)^2+(y-7)^2}$

$9\left[(x+4)^2+(y-2)^2\right] = 4\left[x^2+(y-7)^2\right]$

$9(x^2+8x+y^2-4y+20) = 4(x^2+y^2-14y+49)$

$9x^2+72x+9y^2-36y+180 = 4x^2+4y^2-56y+196$

$5x^2+5y^2+72x+20y-16 = 0$

2 (a) $AM = 3BM$

$\sqrt{(x+5)^2+(y-3)^2} = 3\sqrt{(x+2)^2+(y-1)^2}$

$(x+5)^2+(y-3)^2 = 9[(x+2)^2+(y-1)^2]$

$x^2+10x+25+y^2-6y+9 = 9x^2+36x+36+9y^2-18y+9$

$8x^2+8y^2+26x-12y+11 = 0$

(b) Intersects with the x-axis $\Rightarrow y = 0$

$8x^2+26x+11 = 0$

$(4x+11)(2x+1) = 0$

$x = -\frac{11}{4}, x = -\frac{1}{2}$

Intersection point $= \left(-\frac{11}{4}, 0\right), \left(-\frac{1}{2}, 0\right)$

3 (a) $\sqrt{(x-3)^2+(y+4)^2} = 2\sqrt{(x+2)^2+(y+4)^2}$

$(x-3)^2+(y+4)^2 = 4\left[(x+2)^2+(y+4)^2\right]$

$x^2-6x+9+y^2+8y+16 = 4x^2+16x+16+4y^2+32y+64$

$3x^2+3y^2+22x+24y+55 = 0$

(b) Intersects with the y-axis, $x = 0$

$3y^2+24y+55 = 0$

$b^2-4ac = (24)^2-4(3)(55)$

$= -84 < 0$

Thus, the locus of P does not intersect with the y-axis.

SPM Practice

Paper 1

1 (a) $M = \left(\frac{3+12}{2}, \frac{6+5}{2}\right) = \left(\frac{15}{2}, \frac{11}{2}\right)$

(b) $M = \left(\frac{1(3) + 2(12)}{3}, \frac{1(6) + 2(5)}{3}\right) = \left(9, \frac{16}{3}\right)$

2 $M = (-2, 1) = \left(\frac{-h-1}{2}, \frac{-5+k}{2}\right)$

$$-2 = \frac{-h-1}{2} \qquad 1 = \frac{-5+k}{2}$$
$$-h - 1 = -4 \qquad -5 + k = 2$$
$$h = 3 \qquad k = 7$$

3 (a) $(3, -p) = \left(\frac{-2m+5n}{m+n}, \frac{7m-2n}{m+n}\right)$

$$\frac{-2m+5n}{m+n} = 3$$
$$-2m + 5n = 3m + 3n$$
$$2n = 5m$$
$$\frac{m}{n} = \frac{2}{5}$$
$\therefore AP : PQ = 2 : 5$

(b) $\frac{7m-2n}{m+n} = -p$

$$\frac{7m-5m}{m+\left(\frac{5}{2}m\right)} = -p$$
$$p = -\frac{2m}{\frac{7m}{2}}$$
$$p = -\frac{4}{7}$$

4 Area of triangle $= \frac{1}{2}\begin{vmatrix} -3 & 10 & 6 & -3 \\ 3 & 2 & 10 & 3 \end{vmatrix}$

$$= \frac{1}{2}|(-6 + 100 + 18) - (30 + 12 - 30)|$$
$$= \frac{1}{2}|112 - 12|$$
$$= 50 \text{ units}^2$$

5 Area = 0 (Collinear)

$$\frac{1}{2}\begin{vmatrix} 3 & 4 & -5 & 3 \\ -k & 3k & 6 & -k \end{vmatrix} = 0$$
$$|(9k + 24 + 5k) - (-4k - 15k + 18)| = 0$$
$$|(14k + 24) - (-19k + 18)| = 0$$
$$|33k + 6| = 0$$
$$k = -\frac{2}{11}$$

6 $PQ: y = ax + 2 \Rightarrow m = a$

$RS: by - 6x = ay + 12$
$$ay - by = -6x - 12$$
$$y(a - b) = -6x - 12$$
$$y = -\frac{6}{a-b}x - \frac{12}{a-b}$$
$\therefore m = -\frac{6}{a-b}$

Perpendicular: $m_{PQ} \times m_{RS} = -1$
$$a \times \left(-\frac{6}{a-b}\right) = -1$$
$$-6a = -a + b$$
$$-5a = b$$
$$a = -\frac{1}{5}b$$

7 $\frac{y}{8} - \frac{x}{4} = 1$

$M = (x, 0) \Rightarrow -\frac{x}{4} = 1$
$$x = -4$$

Gradient of AB: $\frac{y}{8} = \frac{x}{4} + 1$
$$y = 2x + 8$$
$$m = 2$$
Perpendicular: $m = -\frac{1}{2}$
$$\frac{y-0}{x+4} = -\frac{1}{2}$$
$$2y = -x - 4$$
$$y = -\frac{1}{2}x - 2$$

8 $2(3x + y) = x - y$
$$6x + 2y = x - y$$
$$3y = -5x \Rightarrow m = -\frac{5}{3}$$
$$\frac{y-7}{x+1} = -\frac{5}{3}$$
$$y - 7 = -\frac{5}{3}x - \frac{5}{3}$$
$$y = -\frac{5}{3}x + \frac{16}{3}$$

9 $\sqrt{(x-1)^2 + (y+3)^2} = \sqrt{(x-6)^2 + (y-5)^2}$
$$(x-1)^2 + (y+3)^2 = (x-6)^2 + (y-5)^2$$
$$x^2 - 2x + 1 + y^2 + 6y + 9 = x^2 - 12x + 36 + y^2 - 10y + 25$$
$$10x + 16y - 51 = 0$$
Locus of P is a straight line.

10 $\frac{QA}{QB} = \frac{2}{5} \Rightarrow 5QA = 2QB$
$$5\sqrt{(x+2)^2 + (y+1)^2} = 2\sqrt{(x-4)^2 + (y-1)^2}$$
$$25\left[(x+2)^2 + (y+1)^2\right] = 4\left[(x-4)^2 + (y-1)^2\right]$$
$$25(x^2 + 4x + 4 + y^2 + 2y + 1) = 4(x^2 - 8x + 16 + y^2 - 2y + 1)$$
$$25x^2 + 100x + 125 + 25y^2 + 50y = 4x^2 - 32x + 4y^2 - 8y + 68$$
$$21x^2 + 21y^2 + 132x + 58y + 57 = 0$$

Paper 2

1 (a) $(-1, h) = \left(\frac{\frac{1}{2}m - 2n}{m+n}, \frac{-5m - \frac{3}{2}n}{m+n}\right)$

(i) $\frac{\frac{1}{2}m - 2n}{m+n} = -1$
$$\frac{1}{2}m - 2n = -m - n$$
$$\frac{3}{2}m = n$$
$$\frac{m}{n} = \frac{2}{3}$$
$\therefore m : n = 2 : 3$

(ii) $\frac{-5m - \frac{3}{2}n}{m+n} = h$
$$\frac{-5\left(\frac{2}{3}\right)n - \frac{3}{2}n}{\frac{2}{3}n + n} = h$$
$$\frac{-\frac{10}{3}n - \frac{3}{2}n}{\frac{5}{3}n} = h$$
$$\frac{-\frac{29}{6}}{\frac{5}{3}} = h$$
$$h = -\frac{29}{10}$$

(b) (i) M = Midpoint of AB

$$(3, 5) = \left(\frac{x+8}{2}, \frac{2+y}{2}\right)$$

$$\frac{x+8}{2} = 3 \qquad \frac{2+y}{2} = 5$$

$$x + 8 = 6 \qquad y + 2 = 10$$

$$x = -2 \qquad y = 8$$

(ii) $Q(r, s)$

$$(3, 5) = \left(\frac{3r+8}{5}, \frac{3s+20}{5}\right)$$

$$\frac{3r+8}{5} = 3 \qquad \frac{3s+20}{5} = 5$$

$$3r + 8 = 15 \qquad 3s + 20 = 25$$

$$3r = 7 \qquad 3s = 5$$

$$r = \frac{7}{3} \qquad s = \frac{5}{3}$$

$$\therefore Q\left(\frac{7}{3}, \frac{5}{3}\right)$$

2 (a) (i) $m_{AB} = \dfrac{-6-2}{0-(-3)} = -\dfrac{8}{3}$

$$m_{BC} = \frac{k-2}{8-(-3)} = \frac{k-2}{11}$$

$$m_{AB} \times m_{BC} = -1$$

$$-\frac{8}{3} \times \left(\frac{k-2}{11}\right) = -1$$

$$\frac{k-2}{11} = \frac{3}{8}$$

$$k - 2 = \frac{33}{8}$$

$$k = \frac{49}{8}$$

(ii) $m_{BC} = \dfrac{3}{8}$

$$\frac{y-2}{x+3} = \frac{3}{8}$$

$$8y - 16 = 3x + 9$$

$$8y = 3x + 25$$

$$y = \frac{8}{3}x + \frac{25}{8}$$

(iii) Area of $\triangle = \dfrac{1}{2}\begin{vmatrix} 0 & -3 & 8 & 0 \\ -6 & 2 & \frac{49}{8} & -6 \end{vmatrix}$

$$= \frac{1}{2}\left|\left(-\frac{147}{8} - 48\right) - (18 + 16)\right|$$

$$= \frac{1}{2}\left|-\frac{803}{8}\right|$$

$$= \frac{803}{16} \text{ units}^2$$

(b) (i)

$$PM = AM$$

$$\sqrt{(x+1)^2 + (y+3)^2} = \sqrt{(2+1)^2 + (5+3)^2}$$

$$(x+1)^2 + (y+3)^2 = 9 + 64$$

$$x^2 + 2x + 1 + y^2 + 6y + 9 = 73$$

$$x^2 + y^2 + 2x + 6y = 63$$

(ii) $(k, -5)$, $k^2 + 25 + 2k - 30 = 63$

$$k^2 + 2k - 68 = 0$$

$$k = -9.31, k = 7.31$$

$$k < 0, \therefore k = -9.31$$

3 (a) $m_{AC} = \dfrac{3-1}{1-7} = -\dfrac{1}{3}$

$m_{BD} = 3$

M = Midpoint of AC

$$= \left(\frac{7+1}{2}, \frac{1+3}{5}\right)$$

$$= (4, 2)$$

Equation of BD: $\dfrac{y-2}{x-4} = 3$

$$y - 2 = 3x - 12$$

$$y = 3x - 10$$

(b) $B(x, 0)$

$0 = 3x - 10$

$x = \dfrac{10}{3}$

$\therefore B\left(\dfrac{10}{3}, 0\right)$

$$(4, 2) = \left(\frac{2x + \frac{50}{3}}{7}, \frac{2y}{7}\right)$$

$$\frac{2y}{7} = 2 \qquad \frac{2x + \frac{50}{3}}{7} = 4$$

$$y = 7 \qquad 2x + \frac{50}{3} = 28$$

$$2x = \frac{34}{3}$$

$$x = \frac{17}{3}$$

$\therefore D\left(\dfrac{17}{3}, 7\right)$

(c) Area $= \dfrac{1}{2}\begin{vmatrix} 7 & \frac{10}{3} & 1 & \frac{17}{3} & 7 \\ 1 & 0 & 3 & 7 & 1 \end{vmatrix}$

$$= \frac{1}{2}\left|\left(10 + 7 + \frac{17}{3}\right) - \left(\frac{10}{3} + 17 + 49\right)\right|$$

$$= \frac{1}{2}\left|-\frac{140}{3}\right|$$

$$= \frac{70}{3} \text{ units}^2$$

(d)

$$AP = 2PC$$

$$\sqrt{(x-7)^2 + (y-1)^2} = 2\sqrt{(x-1)^2 + (y-3)^2}$$

$$(x-7)^2 + (y-1)^2 = 4\left[(x-1)^2 + (y-3)^2\right]$$

$$x^2 - 14x + 49 + y^2 - 2y + 1 = 4x^2 - 8x + 4 + 4y^2 - 24y + 36$$

$$3x^2 + 3y^2 + 6x - 22y - 10 = 0$$

Intersect with the x-axis, $y = 0$

$3x^2 + 6x - 10 = 0$

$b^2 - 4ac = 6^2 - 4(3)(-10)$

$= 156 > 0$

Thus, the locus of P intersects the x-axis.