# Fully-Worked Solutions

## FORM 4

### CHAPTER 5 Progressions

**Self Test 1**

1 $T_2 - T_1 = T_3 - T_2$

$\log_{10} k - \log_{10} 3 = \log_{10} 12 - \log_{10} k$

$\log_{10} \frac{k}{3} = \log_{10} \frac{12}{k}$

$\frac{k}{3} = \frac{12}{k}$

$k^2 = 36$

$k = 6$

$T_{10} = a + 9d$
$= \log_{10} 3 + 9(\log_{10} 6 - \log_{10} 3)$
$= \log_{10} 3 + 9(\log_{10} 3 + \log_{10} 2 - \log_{10} 3)$
$= \log_{10} 3 + 9\log_{10} 2$
$= \log_{10}(3 \times 2^9)$
$= \log_{10} 1\,536$

2 (a) $a = -3, T_n = 25, S_n = 1\,837$

$T_n = -3 + (n - 1)d$

$25 = -3 + (n - 1)d$

$(n - 1)d = 28$ .......... ①

$S_n = \frac{n}{2}[-3 + 25]$

$1\,837 = \frac{n}{2}(22)$

$n = 167$

Thus, there are 167 terms.

(b) From ①,

$(n - 1)d = 28$

$166d = 28$

$d = \frac{14}{83}$

3 $S_6 = 33$

$\frac{6}{2}(2a + 5d) = 33$

$2a + 5d = 11$ .......... ①

$S_{20} = 2\,490$

$\frac{20}{2}(2a + 19d) = 2\,490$

$2a + 19d = 249$ .......... ②

② − ①: $14d = 238$

$d = 17$

$2a + 5(17) = 11$

$2a = -74$

$a = -37$

**Self Test 2**

1 Geometric progression: $9, x, y$

Perimeter $= 9 + x + y$

$37 = 9 + x + y$

$x + y = 28$

$y = 28 - x$

Therefore, it is a geometric progression with terms $9, x, 28 - x$.

$\frac{x}{9} = \frac{28 - x}{x}$

$x^2 = 252 - 9x$

$x^2 + 9x - 252 = 0$

$(x - 12)(x + 21) = 0$

$\therefore x = 12$ cm

$y = 28 - 12 = 16$ cm

Therefore, the length of the other two sides are 12 cm and 16 cm.

2 $-5, 25, -125, 625, \ldots, -78\,125$

$a = -5, r = -\frac{25}{5} = -5$

$T_n = ar^{n-1}$

$-78\,125 = (-5)(-5)^{n-1}$

$(-5)^{n-1} = 15\,625$

$= (-5)^6$

$n - 1 = 6$

$n = 7$

$\therefore S_7 = \frac{a(1 - r^n)}{1 - r}$

$= \frac{(-5)[1 - (-5)^7]}{1 - (-5)}$

$= -65\,105$

3 (a) Arithmetic progression: $T_1, T_3, T_6$ = Geometric progression

Geometric progression: $a, a + 2d, a + 5d$

$a + a + 2d + a + 5d = 57$

$3a + 7d = 57$ .......... ①

$\frac{a + 2d}{a} = \frac{a + 5d}{a + 2d}$

$(a + 2d)^2 = a^2 + 5ad$

$a^2 + 4ad + 4d^2 = a^2 + 5ad$

$ad = 4d^2$

$ad - 4d^2 = 0$

$d(a - 4d) = 0$

$d \neq 0, \therefore a = 4d$ .......... ②

Substitute ② into ①,

$3(4d) + 7d = 57$

$12d + 7d = 57$

$19d = 57$

$d = 3$

$a = 4(3) = 12$

Thus, the three terms are 12, 18 and 27.

(b) $S_3 : S_\infty = 19 : 27$

$\frac{a(1 - r^3)}{1 - r} : \frac{a}{1 - r} = 19 : 27$

$1 - r^3 = \frac{19}{27}$

$r^3 = \frac{8}{27}$

$r^3 = \left(\frac{2}{3}\right)^3$

$\therefore r = \frac{2}{3}$

**SPM Practice**

**Paper 1**

1 (a) $2x - 3, x + 5, 4x + 3$

$(x + 5) - (2x - 3) = (4x + 3) - (x + 5)$

$-x + 8 = 3x - 2$

$4x = 10$

$x = \frac{5}{2}$

(b) $-6, -13, -20, -27, \ldots$

$a = -6, d = -7$

$$\begin{aligned} T_n &= -111 \\ -6 + (n-1)(-7) &= -111 \\ (n-1)(-7) &= -105 \\ n-1 &= 15 \\ n &= 16 \end{aligned}$$

Thus, the $16^{th}$ term $= -111$

**2** $T_5 = 8, T_{10} = -7$

(a) $a + 4d = 8$ ............ ①

$a + 9d = -7$ .......... ②

② − ①: $5d = -15$

$d = -3$

$$\begin{aligned} a &= 8 - 4d \\ &= 8 - 4(-3) \\ &= 20 \end{aligned}$$

$$\begin{aligned} \therefore T_{20} &= a + 19d \\ &= 20 + 19(-3) \\ &= -37 \end{aligned}$$

(b) $d = 150\,000, a = 237\,500$

$$\begin{aligned} T_{10} &= 237\,500 + 9(150\,000) \\ &= \text{RM}1\,587\,500 \end{aligned}$$

**3** $18, 42, 66, \ldots$

$a = 18, d = 42 - 18 = 24$

(a) $$\begin{aligned} S_{15} &= \frac{15}{2}[2(18) + 14(24)] \\ &= \frac{15}{2}(372) \\ &= 2\,790 \end{aligned}$$

(b) $$\begin{aligned} S_{20} - S_6 &= \frac{20}{2}[2(18) + 19(24)] - \frac{6}{2}[2(18) + 5(24)] \\ &= 10(492) - 3(156) \\ &= 4\,452 \end{aligned}$$

**4** $\frac{1}{2}, \frac{2}{3}, \frac{8}{9}, \ldots$

$a = \frac{1}{2}, r = \dfrac{\frac{2}{3}}{\frac{1}{2}} = \frac{4}{3}$

(a) $$\begin{aligned} T_{25} &= \left(\frac{1}{2}\right)\left(\frac{4}{3}\right)^{24} \\ &= 498.31 \end{aligned}$$

(b) $$\begin{aligned} S_{10} &= \frac{\frac{1}{2}\left[\left(\frac{4}{3}\right)^{10} - 1\right]}{\frac{4}{3} - 1} \\ &= \frac{\frac{1}{2}}{\frac{1}{3}}\left[\left(\frac{4}{3}\right)^{10} - 1\right] \\ &= 25.14 \end{aligned}$$

**5** $S_2 = 8$

$a + ar = 8$ ........... ①

$$\begin{aligned} T_3 + T_4 &= ar^2 + ar^3 \\ \frac{49}{2} &= ar^2 + ar^3 \\ r^2(a + ar) &= \frac{49}{2} \\ r^2(8) &= \frac{49}{2} \\ r^2 &= \frac{49}{16} \\ r &= \pm\frac{7}{4} \end{aligned}$$

$r > 0, \therefore r = \frac{7}{4}$

From ①, $a + a\left(\frac{7}{4}\right) = 8$

$$\begin{aligned} \frac{11a}{4} &= 8 \\ a &= \frac{32}{11} \end{aligned}$$

**6** (a) $81, -27, 9, -3, \ldots$

$a = 81, r = -\frac{27}{81} = -\frac{1}{3}$

$$\begin{aligned} S_\infty &= \frac{a}{1-r} \\ &= \frac{81}{1 - \left(-\frac{1}{3}\right)} \\ &= \frac{243}{4} \\ &= 60\frac{3}{4} \end{aligned}$$

(b) $0.354354\ldots = 0.354 + 0.000354 + \ldots$

$a = 0.354, r = \frac{0.000354}{0.354} = 0.001$

$$\begin{aligned} S_\infty &= \frac{0.354}{1 - 0.001} \\ &= \frac{354}{999} \\ &= \frac{118}{333} \end{aligned}$$

**7** $a = 21, T_4 = \frac{21}{8}$

(a) $$\begin{aligned} ar^3 &= \frac{21}{8} \\ 21(r^3) &= \frac{21}{8} \\ r^3 &= \frac{1}{8} \\ r^3 &= \left(\frac{1}{2}\right)^3 \\ r &= \frac{1}{2} \end{aligned}$$

(b) $$\begin{aligned} S_\infty &= \frac{21}{1 - \frac{1}{2}} \\ &= 42 \end{aligned}$$

## Paper 2

**1** (a) $-54, -52\frac{1}{2}, -51, \ldots$

$a = -54, d = \frac{3}{2}$

(i) $$\begin{aligned} S_{15} &= \frac{15}{2}\left[2(-54) + 14\left(\frac{3}{2}\right)\right] \\ &= \frac{15}{2}(-87) \\ &= -652.5 \end{aligned}$$

(ii) $$\begin{aligned} S_{13} - S_4 &= \frac{13}{2}\left[2(-54) + 12\left(\frac{3}{2}\right)\right] - \frac{4}{2}\left[2(-54) + 3\left(\frac{3}{2}\right)\right] \\ &= \frac{13}{2}(-90) - 2\left(-\frac{207}{2}\right) \\ &= -585 + 207 \\ &= -378 \end{aligned}$$

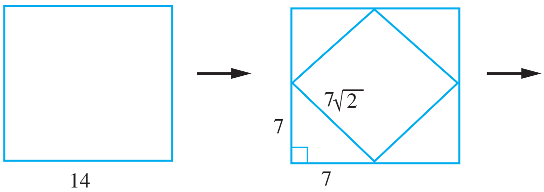
(iii) $S_n > 250$

$$\frac{n}{2}\left[2(-54) + (n-1)\left(\frac{3}{2}\right)\right] > 250$$
$$n\left(-108 + \frac{3}{2}n - \frac{3}{2}\right) > 500$$
$$-108n + \frac{3}{2}n^2 - \frac{3}{2}n > 500$$
$$-216n + 3n^2 - 3n > 1\,000$$
$$3n^2 - 219n - 1\,000 > 0$$

$n < -4.31$ (not accepted), $n > 77.31$

$\therefore n = 78$

(b)




Areas of the squares: $14^2, (7\sqrt{2})^2, \ldots = 196, 98, 49, \ldots$

$$\frac{T_3}{T_2} = \frac{49}{98} = \frac{1}{2}, \frac{T_2}{T_1} = \frac{98}{196} = \frac{1}{2}$$

Thus, the areas of the squares form a geometric progression with $r = \frac{1}{2}$.

**2** (a) 30, $x$, 134

(i) $x - 30 = 134 - x$
$$2x = 164$$
$$x = 82$$

(ii) $\frac{x}{30} = \frac{134}{x}$
$$x^2 = 4\,020$$
$$x = \sqrt{4\,020}$$
$$x = 2\sqrt{1\,005}$$

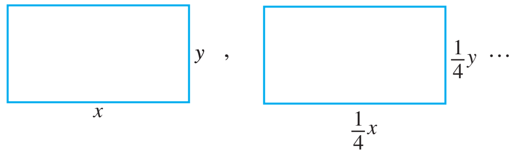
(b) $T_4 = 6, T_7 = -48$

(i) $ar^3 = 6$ .............. ①
$ar^6 = -48$........... ②

$\frac{②}{①}: r^3 = -8$
$$= (-2)^3$$
$$r = -2$$
$$a(-2)^3 = 6$$
$$a = -\frac{6}{8} = -\frac{3}{4}$$

(ii) $T_{11} = ar^{10}$
$$= \left(-\frac{3}{4}\right)(-2)^{10}$$
$$= -768$$

**3**




(a) Area $= xy, \frac{1}{4}x\left(\frac{1}{4}y\right), \frac{1}{16}x\left(\frac{1}{16}y\right), \ldots$
$$= xy, \frac{1}{16}xy, \frac{1}{256}xy, \ldots$$

$$\frac{T_3}{T_2} = \frac{\frac{1}{256}xy}{\frac{1}{16}xy} = \frac{1}{16} \quad , \quad \frac{T_2}{T_1} = \frac{\frac{1}{16}xy}{xy} = \frac{1}{16}$$

$$\frac{T_3}{T_2} = \frac{T_2}{T_1}$$

Thus, the areas of rectangles form a geometric progression.

(b) $x = 45, y = 15$

$45(15), \frac{1}{16}(45)(15), \frac{1}{256}(45)(15), \ldots$

$675, \frac{675}{16}, \frac{675}{256}, \ldots$

(i) $T_n < 30$
$$675\left(\frac{1}{16}\right)^{n-1} < 30$$
$$\left(\frac{1}{16}\right)^{n-1} < 0.044$$
$$(n-1)\lg\left(\frac{1}{16}\right) < \lg(0.044)$$
$$n - 1 > \frac{-1.357}{-1.204}$$
$$n - 1 > 1.127$$
$$n > 2.127$$

$\therefore n = 3$

Thus, the third rectangle has an area of less than 30 $cm^2$.

(ii) $S_\infty = \frac{675}{1 - \frac{1}{16}}$
$$= 720 \text{ cm}^2$$