Fully-Worked Solutions

FORM 4

CHAPTER 3 Systems of Equations

Self Test 1

1 (a) $x + 3y - 2z = 3$ .... ①
$2x - 3y - 8z = 2$ .... ②
$4x - 7y + 2z = 22$ .... ③
①+③: $5x - 4y = 25$ .... ④
4×①: $4x + 12y - 8z = 12$ .... ⑤
⑤−②: $2x + 15y = 10$ .... ⑥
④×2: $10x - 8y = 50$ .... ⑦
⑥×5: $10x + 75y = 50$ .... ⑧
⑦−⑧: $-83y = 0$
$y = 0$
Substitute $y = 0$ into ⑥,
$2x + 15(0) = 10$
$x = 5$
Substitute $x = 5$ and $y = 0$ into ①,
$5 + 3(0) - 2z = 3$
$-2z = -2$
$z = 1$

(b) $x + y = 7$ .... ①
$y - z = -1$ .... ②
$x + 3z = 18$ .... ③
①−②: $x + z = 8$ .... ④
③−④: $2z = 10$
$z = 5$
Substitute $z = 5$ into ②,
$y - 5 = -1$
$y = 4$
Substitute $y = 4$ into ①,
$x + 4 = 7$
$x = 3$

2 $3x = 5y - 4z - 12$
$3x - 5y + 4z = -12$ .... ①
$6y = 20 + 2x + 8z$
$2x - 6y + 8z = -20$
$x - 3y + 4z = -10$ .... ②
$4x = y + 8z + 6$
$4x - y - 8z = 6$ .... ③
①−②: $2x - 2y = -2$
$x - y = -1$ .... ④
②×2: $2x - 6y + 8z = -20$ .... ⑤
⑤+③: $6x - 7y = -14$ .... ⑥
From ④, $x = y - 1$ .... ⑦
Substitute ⑦ into ⑥,
$6(y - 1) - 7y = -14$
$6y - 6 - 7y = -14$
$-y = -8$
$y = 8$
From ⑦,
$x = 8 - 1 = 7$
Substitute $x = 7$ and $y = 8$ into ②,
$7 - 3(8) + 4z = -10$
$7 - 24 + 4z = -10$
$4z = 7$
$z = \frac{7}{4}$

Thus, the intersection point is $\left(7, 8, \frac{7}{4}\right)$.

3 $5x + 2y - 3z = 6$ .... ①
$x + y + z = 6$ .... ②
$\left(\frac{5}{16}x + \frac{1}{8}y - \frac{3}{16}z = 9\right) \times 16$
$5x + 2y - 3z = 144$ .... ③
①−③: $0x + 0y + 0z = -138$ (no solution)
Thus, the three planes do not intersect at any point.

Self Test 2

1 $3x + 2y + 3 = 4$
$2y = 1 - 3x$
$y = \frac{1 - 3x}{2}$ .... ①
$3x^2 - 2y^2 + 9 = 4$
$3x^2 - 2y^2 = -5$ .... ②
Substitute ① into ②,
$3x^2 - 2\left(\frac{1 - 3x}{2}\right)^2 = -5$
$3x^2 - \frac{(1 - 3x)^2}{2} = -5$
$6x^2 - (1 - 3x)^2 = -10$
$6x^2 - 1 + 6x - 9x^2 = -10$
$-3x^2 + 6x + 9 = 0$
$x^2 - 2x - 3 = 0$
$(x + 1)(x - 3) = 0$
$x = -1, x = 3$
$x = 3, y = \frac{1 - 3(3)}{2} = -4$
$x = -1, y = \frac{1 - 3(-1)}{2} = 2$

2 $k - 3p = 1$
$k = 1 + 3p$ .... ①
$p + pk - 2k = 0$ .... ②
Substitute ① into ②,
$p + p(1 + 3p) - 2(1 + 3p) = 0$
$p + p + 3p^2 - 2 - 6p = 0$
$3p^2 - 4p - 2 = 0$
$p = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-2)}}{2(3)}$
$p = -0.387, p = 1.72$
$p = -0.387, k = 1 + 3(-0.387) = -0.161$
$p = 1.72, k = 1 + 3(1.72) = 6.16$

3 Length of arc AD = Length of arc CB
$= \frac{1}{2} \times 2 \times \frac{22}{7} \times \left(\frac{7}{2}x\right)$
$= 11x$ cm
Perimeter $= AB + BC + AD + CD$
$42 = 11x + 11x + y + y$
$22x + 2y = 42$
$11x + y = 21$
$y = 21 - 11x$ .... ①

Area = 63
$y(7x) + \frac{22}{7}\left(\frac{7x}{2}\right)^2 = 63$
$7xy + \frac{22}{7}\left(\frac{49x^2}{4}\right) = 63$
$7xy + \frac{77x^2}{2} = 63$
$14xy + 77x^2 = 126$ .... ②

Substitute ① into ②,

$$14x(21-11x)+77x^2=126$$
$$2x(21-11x)+11x^2=18$$
$$42x-22x^2+11x^2=18$$
$$42x-22x^2+11x^2-18=0$$
$$-11x^2+42x-18=0$$
$$11x^2-42x+18=0$$
$$x=\frac{-(-42)\pm\sqrt{(-42)^2-4(11)(18)}}{2(11)}$$

$x=0.492, 3.326$

$x=3.326$,

$y=21-11(3.326)=-15.586$ (not accepted)

Thus, $x=0.492$, $y=21-11(0.492)=15.588$

SPM Practice

Paper 1

1 $3x-2y=-8$ .... ①

$y-x+z=6$ .... ②

$-4x+3y-2z=5$ .... ③

$2\times$②: $-2x+2y+2z=12$ .... ④

④ + ③: $-6x+5y=17$ .... ⑤

$2\times$①: $6x-4y=-16$ .... ⑥

⑥ + ⑤: $y=1$

Substitute $y=1$ into ①,

$$3x-2=-8$$
$$3x=-6$$
$$x=-2$$

Substitute $x=-2$ and $y=1$ into ②,

$$1-(-2)+z=6$$
$$3+z=6$$
$$z=3$$

2 $x+y+z=4$ .... ①

$2x+3y+z=9$ .... ②

$3x+2y+3z=10$ .... ③

② − ①: $x+2y=5$ .... ④

$3\times$①: $3x+3y+3z=12$ .... ⑤

⑤ − ③: $y=2$

From ④, $x+2(2)=5$

$x=1$

From ①, $x+y+z=4$

$1+2+z=4$

$z=1$

∴ Intersection point = (1, 2, 1)

3 $2x+3y=12$

$y=\dfrac{12-2x}{3}$ .... ①

$x^2+3y^2=28$ .... ②

Substitute ① into ②,

$$x^2+3\left(\frac{12-2x}{3}\right)^2=28$$
$$x^2+\frac{(12-2x)^2}{3}=28$$
$$3x^2+(12-2x)^2=84$$
$$3x^2+144-48x+4x^2-84=0$$
$$7x^2-48x+60=0$$
$$x=\frac{-(-48)\pm\sqrt{(-48)^2-4(7)(60)}}{2(7)}$$

$x=1.64$, $x=5.21$

$x=1.64$, $y=\dfrac{12-2(1.64)}{3}=2.91$

$x=5.21$, $y=\dfrac{12-2(5.21)}{3}=0.527$

4 $\dfrac{x}{2}+\dfrac{y}{3}=4$

$$x=2\left(4-\frac{y}{3}\right)$$

$x=8-\dfrac{2y}{3}$ .... ①

$2xy=45$ .... ②

Substitute ① into ②,

$$2\left(8-\frac{2y}{3}\right)y=45$$
$$16y-\frac{4y^2}{3}=45$$
$$48y-4y^2=135$$
$$4y^2-48y+135=0$$
$$(2y-9)(2y-15)=0$$
$$y=\frac{9}{2}, y=\frac{15}{2}$$

$y=\dfrac{9}{2}$, $x=8-\dfrac{2\left(\frac{9}{2}\right)}{3}=5$

$y=\dfrac{15}{2}$, $x=8-\dfrac{2\left(\frac{15}{2}\right)}{3}=3$

$\therefore \left(3, \dfrac{15}{2}\right), \left(5, \dfrac{9}{2}\right)$

5 $4x+y-5=0$

$y=5-4x$ .... ①

$27x^2+21xy+2y^2=0$ .... ②

Substitute ① into ②,

$$27x^2+21x(5-4x)+2(5-4x)^2=0$$
$$27x^2+105x-84x^2+2(25-40x+16x^2)=0$$
$$27x^2+105x-84x^2+50-80x+32x^2=0$$
$$-25x^2+25x+50=0$$
$$x^2-x-2=0$$
$$(x+1)(x-2)=0$$
$$x=-1,\ x=2$$

$x=-1$, $y=5-4(-1)=9$

$x=2$, $y=5-4(2)=-3$

6 $2y=x+2$

$x=2y-2$ .... ①

$x(1-y)+y=0$

Substitute ① into ②,

$$(2y-2)(1-y)+y=0$$
$$2y-2y^2-2+2y+y=0$$
$$-2y^2+5y-2=0$$
$$2y^2-5y+2=0$$
$$(2y-1)(y-2)=0$$
$$y=\frac{1}{2}, y=2$$

$y=\dfrac{1}{2}$, $x=2\left(\dfrac{1}{2}\right)-2=-1$

$y=2$, $x=2(2)-2=2$

$\therefore P\left(-1, \dfrac{1}{2}\right), Q(2, 2)$

7 $(1, 2)\Rightarrow a(1)+b(2)=2$

$a=2-2b$ .... ①

$(1, 2)\Rightarrow b(1)+a^2(2)=10$

$b+2a^2=10$ .... ②

Substitute ① into ②,

$$b+2(2-2b)^2=10$$
$$b+2(4-8b+4b^2)=10$$
$$b+8-16b+8b^2=10$$
$$8b^2-15b-2=0$$
$$(8b+1)(b-2)=0$$

$$b = -\frac{1}{8}, b = 2$$

$$b = -\frac{1}{8}, a = 2 - 2\left(-\frac{1}{8}\right) = \frac{9}{4}$$

$$b = 2, a = 2 - 2(2) = -2$$

8 Let the radius of circle I = x cm and the radius of circle II = y cm

Total circumference = $(2\pi j_1) + 2\pi j_2$

$$2\left(\frac{22}{7}\right)x + 2\left(\frac{22}{7}\right)y = 40$$

$$44x + 44y = 280$$

$$11x + 11y = 70$$

$$x = \frac{70 - 11y}{11} \text{ .... ①}$$

Total area $= \pi j_1^2 + \pi j_2^2$

$$126 = \left(\frac{22}{7}\right)x^2 + \left(\frac{22}{7}\right)y^2$$

$$22x^2 + 22y^2 = 882$$

$$11x^2 + 11y^2 = 441 \text{ .... ②}$$

Substitute ① into ②,

$$11\left(\frac{70 - 11y}{11}\right)^2 + 11y^2 = 441$$

$$\frac{(70 - 11y)^2}{11} + 11y^2 = 441$$

$$4\,900 - 1\,540y + 121y^2 + 121y^2 = 4\,851$$

$$242y^2 - 1\,540y + 49 = 0$$

$$y = 0.032, \ y = 6.33$$

$$y = 0.032, x = \frac{70 - 11(0.032)}{11} = 6.33$$

$$y = 6.33, x = \frac{70 - 11(6.33)}{11} = 0.034$$

Thus, the radii of the two circles are 6.33 cm and 0.03 cm respectively.

9 $y = 3 - x$ .... ①

$xy = \frac{5}{4}$ .... ②

Substitute ① into ②,

$$x(3 - x) = \frac{5}{4}$$

$$4x(3 - x) = 5$$

$$12x - 4x^2 = 5$$

$$4x^2 - 12x + 5 = 0$$

$$(2x - 1)(2x - 5) = 0$$

$$x = \frac{1}{2}, x = \frac{5}{2}$$

$$x = \frac{1}{2}, y = 3 - \left(\frac{1}{2}\right) = \frac{5}{2}$$

$$x = \frac{5}{2}, y = 3 - \left(\frac{1}{2}\right) = \frac{1}{2}$$

Thus, the two numbers are $\frac{1}{2}$ and $\frac{5}{2}$.

10 Let the pocket money of Amanda = RMx, Hazel = RMy and Sathya = RMz

$x + y + z = 95$ .... ①

$z = x + 6$ .... ②

$y = 3z$ .... ③

Substitute ③ into ①,

$x + 3z + z = 95$

$x + 4z = 95$.......... ④

Substitute ② into ④,

$$x + 4(x + 6) = 95$$

$$x + 4x + 24 = 95$$

$$5x = 71$$

$$x = 14.2$$

$$x + 4z = 95$$

$$14.2 + 4z = 95$$

$$4z = 80.8$$

$$z = 20.2$$

$$y = 3(20.2) = 60.6$$

Let the pocket money of Amanda = RM14.20, Hazel = RM60.60 and Sathya = RM20.20

Paper 2

1 Total length of all sides: $4(y + 3) + 4x + 4(x + y) = 20$

$$4y + 12 + 4x + 4x + 4y = 20$$

$$8x + 8y = 8$$

$$x + y = 1$$

$$y = 1 - x \text{ .... ①}$$

Surface area:

$$x(y + 3)(2) + x(x + y)(2) + (y + 3)(x + y)(2) = 10$$

$$2xy + 6x + 2x^2 + 2xy + 2xy + 2y^2 + 6x + 6y = 10$$

$$2x^2 + 2y^2 + 6xy + 12x + 6y = 10$$

$$x^2 + y^2 + 3xy + 6x + 3y = 5 \text{ .... ②}$$

Substitute ① into ②,

$$x^2 + (1 - x)^2 + 3x(1 - x) + 6x + 3(1 - x) = 5$$

$$x^2 + 1 - 2x + x^2 + 3x - 3x^2 + 6x + 3 - 3x = 5$$

$$-x^2 + 4x - 1 = 0$$

$$x^2 - 4x + 1 = 0$$

$$x = 0.268, \ x = 3.732$$

$x = 0.268, y = 1 - 0.268 = 0.732$

$x = 3.732, y = 1 - 3.732 = -2.732$ (not accepted)

$\therefore x = 0.268, y = 0.732$

2 (a) (i) $x + y = 13$

$y = 13 - x$ .... ①

$$x - y = \frac{24}{x}$$

$x(x - y) = 24$ .... ②

Substitute ① into ②,

$$x[x - (13 - x)] = 24$$

$$x(2x - 13) = 24$$

$$2x^2 - 13x - 24 = 0$$

$$(2x + 3)(x - 8) = 0$$

$$x = -\frac{3}{2}, x = 8$$

$$x = -\frac{3}{2}, y = 13 - \left(-\frac{3}{2}\right) = \frac{29}{2}$$

$$x = 8, y = 13 - (8) = 5$$

(ii) The solution $x = -\frac{3}{2}, y = \frac{29}{2}$ is not practical because the mass cannot take negative values.

(b) A system of equations can have unique solution, many solutions or no solution depending on the type of intersection of lines or planes.

3 (a) $2k + r = 14.50$ .... ①

$4r + t = 6.95$ .... ②

$2k + t = 13.45$ .... ③

① − ③: $r - t = 1.05$ .... ④

② + ④: $5r = 8$

$r = 1.6$

Substitute $r = 1.6$ into ④,

$1.6 - t = 1.05$

$t = 0.55$

Substitute $r = 1.6$ into ①,

$2k + 1.6 = 14.50$

$2k = 12.9$

$k = 6.45$

Price of cup of coffee = RM6.45, price of a piece *roti canai* = RM1.60 and price of a boiled egg = RM0.55

(b) $A + B + C = 12\ 000$ .... ①
$0.1A + 0.08B + 0.12C = 1\ 230$
$10A + 8B + 12C = 123\ 000$
$5A + 4B + 6C = 61\ 500$ .... ②
$A + B = C$ .... ③

Substitute ③ into ① and ②,
$A + B + (A + B) = 12\ 000$
$2A + 2B = 12\ 000$
$A + B = 6\ 000$
$B = 6\ 000 - A$ .... ④
$5A + 4B + 6(A + B) = 61\ 500$ .... ⑤

Substitute ④ into ⑤,
$5A + 4(6\ 000 - A) + 6\ (A + 6\ 000 - A) = 61\ 500$
$5A - 4A + 24\ 000 + 36\ 000 = 61\ 500$
$A = 1\ 500$
$B = 6\ 000 - 1\ 500$
$= 4\ 500$
$C = 4\ 500 + 1\ 500$
$= 6\ 000$

∴ Account A = RM1 500, account B = RM4 500 and account C = RM6 000