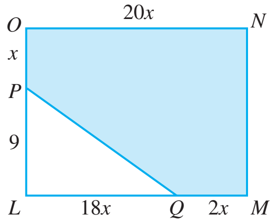
FORM 4

CHAPTER 2 Quadratic Functions

Self Test 1

1 Area of shaded region $= 20x(x+9) - \frac{1}{2}(9)(18x)$

$$119 = 20x^2 + 180x - 81x$$
$$20x^2 + 99x - 119 = 0$$



$$QP = \sqrt{9^2 + (18x)^2}$$
$$= \sqrt{81 + 324x^2}$$
$$= \sqrt{81(1 + 4x^2)}$$
$$= 9\sqrt{1 + 4x^2}$$

Perimeter of QLP

$$= 9 + 18x + 9\sqrt{1 + 4x^2}$$
$$= 9\left(1 + 2x + \sqrt{1 + 4x^2}\right)$$

2 $2x^2 + 2x + 3 = 0$

$$x^2 + x + \frac{3}{2} = 0$$

$$\alpha + \beta = -1,\ \alpha\beta = \frac{3}{2}$$

S.O.R.: $\frac{\alpha}{\beta + 1} + \frac{\beta}{\alpha + 1} = \frac{\alpha^2 + \alpha + \beta + \beta^2}{(\alpha + 1)(\beta + 1)}$

$$= \frac{\alpha^2 + \beta^2 + \alpha + \beta}{\alpha\beta + \alpha + \beta + 1}$$
$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta + (\alpha + \beta)}{\alpha\beta + (\alpha + \beta) + 1}$$
$$= \frac{(-1)^2 - 2\left(\frac{3}{2}\right) + (-1)}{\frac{3}{2} + (-1) + 1}$$
$$= -\frac{3}{\frac{3}{2}}$$
$$= -2$$

P.O.R.: $\left(\frac{\alpha}{\beta + 1}\right)\left(\frac{\beta}{\alpha + 1}\right) = \frac{\alpha\beta}{\alpha\beta + \alpha + \beta + 1}$

$$= \frac{\left(\frac{3}{2}\right)}{\frac{3}{2} + (-1) + 1}$$
$$= 1$$

$\therefore$ New quadratic equation: $x^2 + 2x + 1 = 0$

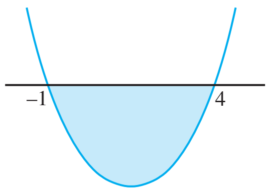
3 $4x^2 - 5x + 7 > 6 - x$

$$4x^2 - 5x + 7 - 6 + x > 0$$
$$4x^2 - 4x + 1 > 0$$
$$(2x - 1)^2 > 0$$

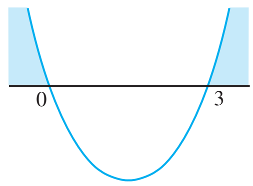
$x \in \mathbb{R}, x \neq \frac{1}{2}$

4 (a) $2 \leqslant f(x) \leqslant 6$

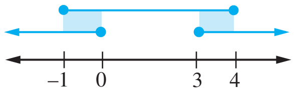
$6 + 3x - x^2 \geqslant 2$	$6 + 3x - x^2 \leqslant 6$
$x^2 - 3x - 4 \leqslant 0$	$x^2 - 3x \geqslant 0$
$(x - 4)(x + 1) \leqslant 0$	$x(x - 3) \geqslant 0$



$-1 \leqslant x \leqslant 4$



$x \leqslant 0, x \geqslant 3$

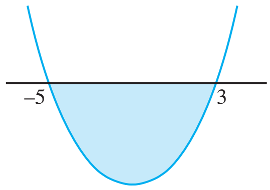


$\therefore -1 \leqslant x \leqslant 0$ or $3 \leqslant x \leqslant 4$

(b) $x^2 + cx - d < 0, -5 < x < 3$

$$(x + 5)(x - 3) < 0$$
$$x^2 + 2x - 15 < 0$$

$\therefore c = 2, d = 15$



Self Test 2

1 $2x - 1 = x^2 - 5x + p$

$$x^2 - 7x + p + 1 = 0$$

$b^2 - 4ac > 0$ (2 different roots, $a \neq b$)

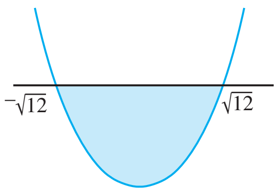
$$(-7)^2 - 4(1)(p + 1) > 0$$
$$49 - 4p - 4 > 0$$
$$-4p > -45$$
$$p < \frac{45}{4}$$

2 $6kx = \frac{3x - 5}{x}$

$$6kx^2 = 3x - 5$$
$$6kx^2 - 3x + 5 = 0$$
$$b^2 - 4ac > 0$$
$$(-3)^2 - 4(6k)(5) > 0$$
$$9 - 120k > 0$$
$$k < \frac{9}{120}$$
$$k < \frac{3}{40}$$

3 $x^2 + k = kx + 2x - 4$

$$x^2 - (k + 2)x + k + 4 = 0$$
$$b^2 - 4ac < 0$$
$$[-(k + 2)]^2 - 4(1)(k + 4) < 0$$
$$k^2 + 4k + 4 - 4k - 16 < 0$$
$$k^2 - 12 < 0$$
$$\left(k + \sqrt{12}\right)\left(k - \sqrt{12}\right) < 0$$
$$-\sqrt{12} < k < \sqrt{12}$$
$$-2\sqrt{3} < k < 2\sqrt{3}$$



Self Test 3

1 $f(x) = tx^2 - 4x + s$

$$f(x) = t\left[x^2 - \frac{4}{t}x + \frac{s}{t}\right]$$

$= t\left[\left(x - \frac{2}{t}\right)^2 - \frac{4}{t^2} + \frac{s}{t}\right]$

$= t\left(x - \frac{2}{t}\right)^2 - \frac{4}{t} + s$

Axis of symmetry: $x = -4 \Rightarrow x = \frac{2}{t}$

$\frac{2}{t} = -4$

$t = -\frac{1}{2}$

Maximum value $= 3$

$-\frac{4}{t} + s = 3$

$-\frac{4}{\left(-\frac{1}{2}\right)} + s = 3$

$s = -5$

2 $f(x) = mx^2 - kx - 10, -4 < m < 2$
The function has minimum point $\Rightarrow m < 0$
m is an integer, thus $m = -2$ (even number)

$f(x) = -2x^2 - kx - 10$

$= -2\left(x^2 + \frac{k}{2}x\right) - 10$

$= -2\left[\left(x + \frac{k}{4}\right)^2 - \frac{k^2}{16}\right] - 10$

$= -2\left(x + \frac{k}{4}\right)^2 + \frac{k^2}{8} - 10$

Maximum value $= -8$

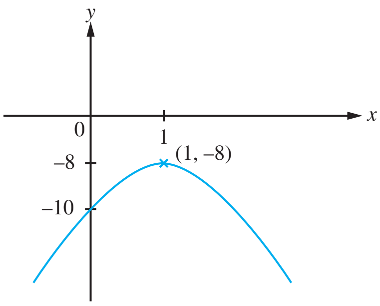
$\frac{k^2}{8} - 10 = -8$

$\frac{k^2}{8} = 2$

$k^2 = 16$

$k = \sqrt{16} = \pm 4$

Since $k < 0$, $\therefore k = -4$
Thus, $f(x) = -2x^2 + 4x - 10$
Maximum point $= \left(-\frac{k}{4}, -8\right) = (1, -8)$
y-intercept $= -10$



Range of $f(x)$: $f(x) \leqslant -8$

3 $f(x) = (x - k)^2 + 15$
(a) Axis of symmetry, $x = -2$
$\therefore k = -2$
(b) Equation of axis of symmetry, $x = -2$
(c) When $x = 0$,
$f(0) = [0 - (-2)]^2 + 15$
$= 4 + 15$
$= 19$
$\therefore A(0, 19)$
(d) $f(x) = (x + 2)^2 + 15$
The function $f(x)$ when its graph reflected on the x-axis is $f(x) = -(x + 2)^2 - 15$.

SPM Practice

Paper 1

1 (a) $3x^2 + 3px - q = 0$

$x^2 + px - \frac{q}{3} = 0$

S.O.R.: $5 + 3 = 8 = -p$

$p = -8$

P.O.R.: $5(3) = -\frac{q}{3}$

$15 = -\frac{q}{3}$

$q = -45$

(b) $3x^2 + 3px - q = k$

$3x^2 + 3(-8)x - (-45) = k$

$3x^2 - 24x + 45 - k = 0$

$b^2 - 4ac < 0$

$(-24)^2 - 4(3)(45 - k) < 0$

$576 - 540 + 12k < 0$

$12k < -36$

$k < -3$

2 (a) $2x^2 + 6x - 1 = 2(x + m)^2 + n$
$2x^2 + 6x - 1 = 2(x^2 + 3x) - 1$

$= 2\left[\left(x + \frac{3}{2}\right)^2 - \frac{9}{4}\right] - 1$

$= 2\left(x + \frac{3}{2}\right)^2 - \frac{9}{2} - 1$

$= 2\left(x + \frac{3}{2}\right)^2 - \frac{11}{2} \ldots ①$

Compare ① with $2(x + m)^2 + n$,

$\therefore m = -\frac{3}{2}, n = -\frac{11}{2}$

(b) $2x^2 + 6x - 1 = 0$

$2\left(x + \frac{3}{2}\right)^2 - \frac{11}{2} = 0$

$\left(x + \frac{3}{2}\right)^2 = \frac{11}{4}$

$x + \frac{3}{2} = \pm\sqrt{\frac{11}{4}}$

$x = \frac{\sqrt{11}}{2} - \frac{3}{2}$ or $x = -\frac{\sqrt{11}}{2} - \frac{3}{2}$

$x = \frac{-\sqrt{11} - 3}{2}$ $\quad$ $x = \frac{\sqrt{11} - 3}{2}$

3 $x^2 - px + 5 = 0$
S.O.R.: $m + n = p \ldots ①$
P.O.R.: $mn = 5 \ldots ②$
$2x^2 + 12x - q = 0$

$x^2 + 6x - \frac{q}{2} = 0$

S.O.R.: $3m + 3n = -6$

$m + n = -2$

From ①, $m + n = p$

$\therefore p = -2$

P.O.R.: $3m \times 3n = -\frac{q}{2}$

$9(5) = -\frac{q}{2}$

$q = -90$

4 $y - kx = 4$

$y = 4 + kx \ldots ①$

$y = x^2 - 4x + 5$

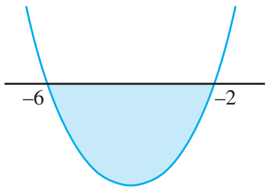
$4 + kx = x^2 - 4x + 5$

$x^2 - kx - 4x + 1 = 0$

$x^2 + (4 + k)x + 1 = 0$

$b^2 - 4ac < 0$ (does not intersect)

$[-(4+k)]^2 - 4(1)(1) < 0$
$16 + 8k + k^2 - 4 < 0$
$k^2 + 8k + 12 < 0$
$(k+6)(k+2) < 0$



$\therefore -6 < k < -2$

5 $y = x^2 + kx + h$

(a) Roots = 1, 5

S.O.R.: $1 + 5 = -k$, $k = -6$

P.O.R.: $1 \times 5 = h$, $h = 5$

(b) $y = x^2 - 6x + 5$
$= (x-3)^2 - 9 + 5$
$= (x-3)^2 - 4$

(i) Graph reflected on the x-axis: $y = -(x-3)^2 + 4$
(ii) Graph reflected on the y-axis: $y = (x+3)^2 - 4$

6 $f(x) = ax^2 + bx + c$
$= a\left(x^2 + \frac{b}{a}x\right) + c$
$= a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right] + c$
$= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$

Axis of symmetry: $x = -\frac{b}{2a}$
$1 = -\frac{b}{2a}$
$-b = 2a$
$b = -2a$ … ①

Intersects the x-axis: (5, 0)
$f(5) = 0$
$a(5)^2 + b(5) + c = 0$
$25a + 5b + c = 0$ … ②
At (2, 5), $f(2) = 5$
$a(2)^2 + b(2) + c = 5$
$4a + 2b + c = 5$ … ③
② – ③:
$21a + 3b = -5$
Substitute $b = -2a$,
$21a + 3(-2a) = -5$
$21a - 6a = -5$
$15a = -5$
$a = -\frac{1}{3}$
Substitute $a = -\frac{1}{3}$ into $b = -2a$,
$b = -2\left(-\frac{1}{3}\right)$
$= \frac{2}{3}$
From ③, $4a + 2b + c = 5$
$4\left(-\frac{1}{3}\right) + 2\left(\frac{2}{3}\right) + c = 5$
$c = 5$
$\therefore f(x) = -\frac{1}{3}x^2 + \frac{2}{3}x + 5$

7 $P = 240x - 3x^2 - 800$
$= -3x^2 + 240x - 800$
$= -3[x^2 - 80x] - 800$
$= -3[(x-40)^2 - 40^2] - 800$
$= -3(x-40)^2 + 4\,800 - 800$
$= -3(x-40)^2 + 4\,000$

(a) Number of refrigerators
$= 40$
(b) Maximum profit
= RM4 000

Alternative method
$x = -\frac{b}{2a}$
$= \frac{240}{2(-3)}$
$= 40$
$P = 240(40) - 3(40)^2 - 800$
$= 4\,000$

Paper 2

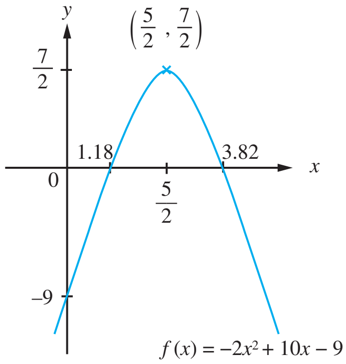
1 (a) Area of triangle $= \frac{1}{2}(x-2)(3x+2)$
$50 = \frac{1}{2}(x-2)(3x+2)$
$(x-2)(3x+2) = 100$
$3x^2 + 2x - 6x - 4 = 100$
$3x^2 - 4x - 104 = 0$ (Shown)
$x = \frac{4 \pm \sqrt{(-4)^2 - 4(3)(-104)}}{2(3)}$
$= \frac{4 \pm \sqrt{1\,264}}{6}$
$= 6.59, -5.26$ (not accepted)

(b) Area of triangle $= \frac{1}{2}(x-2)(3x+2)$
$= \frac{1}{2}(3x^2 - 4x - 4)$
$= \frac{3}{2}x^2 - 2x - 2$
$= \frac{3}{2}\left(x^2 - \frac{4}{3}x\right) - 2$
$= \frac{3}{2}\left[\left(x - \frac{2}{3}\right)^2 - \frac{4}{9}\right] - 2$
$= \frac{3}{2}\left(x - \frac{2}{3}\right)^2 - \frac{2}{3} - 2$
$= \frac{3}{2}\left(x - \frac{2}{3}\right)^2 - \frac{8}{3}$
$\therefore x = \frac{2}{3}$

2 $f(x) = -2x^2 + 10x - 9$
$= -2(x^2 - 5x) - 9$
$= -2\left[\left(x - \frac{5}{2}\right)^2 - \frac{25}{4}\right] - 9$
$= -2\left(x - \frac{5}{2}\right)^2 + \frac{25}{2} - 9$
$= -2\left(x - \frac{5}{2}\right)^2 + \frac{7}{2}$

(a) Maximum value: $f(x) = \frac{7}{2}$
(b) Axis of symmetry: $x = \frac{5}{2}$
(c) Coordinates of maximum point $= \left(\frac{5}{2}, \frac{7}{2}\right)$

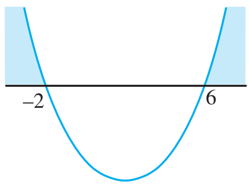
y-intercept: $x = 0$
$f(0) = -9$
$-2\left(x - \frac{5}{2}\right)^2 + \frac{7}{2} = 0$
$\left(x - \frac{5}{2}\right)^2 = \frac{7}{4}$
$x - \frac{5}{2} = \pm 1.323$
$x = -1.323 + 2.5,\ 1.323 + 2.5$
$= 1.177, 3.823$



3 (a) $f(x) = x^2 - px + p + 3$

On the x-axis: $y = 0$

$$x^2 - px + p + 3 = 0$$
$$b^2 - 4ac > 0$$
$$(-p)^2 - 4(1)(p+3) > 0$$
$$p^2 - 4p - 12 > 0$$
$$(p-6)(p+2) > 0$$



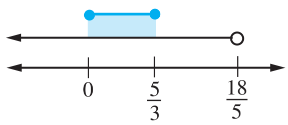
$p < -2, p > 6 \Leftrightarrow p < p_1, p > p_2$

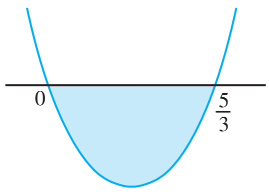
$\therefore p_1 = -2,\ p_2 = 6$

(b) $3x - 1 < 2(5 - x) + 7$

$$3x - 1 < 10 - 2x + 7$$
$$5x < 18$$
$$x < \frac{18}{5}$$

$$3x^2 \leqslant 5x$$
$$3x^2 - 5x \leqslant 0$$
$$x(3x-5) \leqslant 0$$





$\therefore 0 \leqslant x \leqslant \frac{5}{3}$

(c) $4x^2 - 3kx + 5 = 0$

$$x^2 - \frac{3k}{4}x + \frac{5}{4} = 0$$

Let the roots of the equation = α and 3α (1 : 3)

S.O.R.: $\alpha + 3\alpha = \frac{3k}{4}$

$$4\alpha = \frac{3k}{4}$$
$$\alpha = \frac{3k}{16}$$

P.O.R.: $\alpha(3\alpha) = \frac{5}{4}$

$$3\alpha^2 = \frac{5}{4}$$
$$\alpha^2 = \frac{5}{12}$$
$$\alpha = \pm\sqrt{\frac{5}{12}}$$

Since $k > 0$, $\alpha > 0$, thus $\alpha = \sqrt{\frac{5}{12}}$

$$\frac{3k}{16} = \sqrt{\frac{5}{12}}$$
$$k = \frac{\sqrt{5}}{2\sqrt{3}} \times \frac{16}{3}$$
$$= \frac{16\sqrt{5}}{6\sqrt{3}}$$
$$= \frac{8}{3}\sqrt{\frac{5}{3}} \quad \text{or} \quad \frac{8\sqrt{15}}{9}$$

The two roots = $\alpha, 3\alpha$

$$= \frac{\sqrt{5}}{2\sqrt{3}}, 3\left(\frac{\sqrt{5}}{2\sqrt{3}}\right)$$
$$= \frac{\sqrt{15}}{6}, \frac{\sqrt{15}}{2}$$