Fully-Worked Solutions

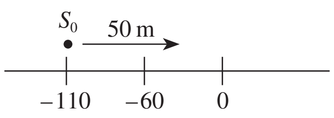
FORM 5

CHAPTER 8 Kinematics of Linear Motion

Self Test 1

1 (a) (i) Initial position = 110 m to the left
$\therefore s = -110$ m

(ii)



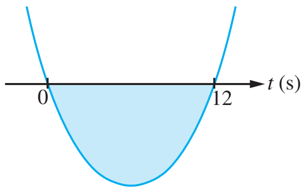
$-110 + 50 = -60$
At 6 seconds, $s = -60$ m

(b) Total distance travelled in the first 10 seconds
$= 50 + 80$
$= 130$ m

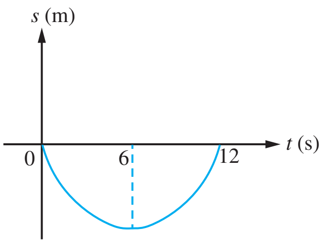
(c) $t = 10, s = -110 + 50 - 80$
$= -140$ m

2 $s = t^2 - 12t$

(a) $s < 0$
$t^2 - 12t < 0$
$t(t - 12) < 0$
$\therefore 0 < t < 12$



(b)



$t = 6, s = 6^2 - 12(6)$
$= -36$ m
$t = 10, s = 10^2 - 12(10)$
$= -20$ m
Total distance $= 36 + (36 - 20) = 52$ m

3 $v = 2t - 8$

(a) $t = 0, v = 2(0) - 8 = -8$ m s^{-1}
Thus, initial velocity is 8 m s^{-1} to the left.

(b) Change direction, $v = 0$
$2t - 8 = 0$
$t = 4$

The object changes direction at the 4th second.

(c) $v = 2(8) - 8 = 8$ m s^{-1}
The object starts moving to the left with a velocity of 8 m s^{-1}. Then, changes the direction of its motion at 4 seconds and moves to the right until it achieves a velocity of 8 m s^{-1} at the 8th second.

Self Test 2

1 (a) $s = 2t^3 - 3t^2 - 6t - 9$
$v = 6t^2 - 6t - 6$

$t = 1, v = 6(1)^2 - 6(1) - 6$
$= -6$ m s^{-1}

At 1 second, velocity = 6 m s^{-1} to the left.
$t = 2, v = 6(2)^2 - 6(2) - 6$
$= 6$ m s^{-1}

At 2 seconds, velocity = 6 m s^{-1} to the right.
$t = 4, v = 6(4)^2 - 6(4) - 6$
$= 66$ m s^{-1}

At 4 seconds, velocity = 66 m s^{-1} to the right.

(b) $v = 6t^2 - 6t - 6$
$a = 12t - 6$
$t = 0, a = 12(0) - 6$
$= -6$ m s^{-2}

2 $s = 5t^2 - 4t - 12$
Passes through O again, $s = 0$
$5t^2 - 4t - 12 = 0$
$(5t + 6)(t - 2) = 0$
$t = 2$
$v = 10t - 4$
$t = 2, v = 10(2) - 4$
$= 16$ m s^{-1}

3 $s = t^3 - 5t^2 + 3t + 11$
$v = 3t^2 - 10t + 3$
$a = 6t - 10$

(a) $t = 4, v = 3(4)^2 - 10(4) + 3$
$= 11$ m s^{-1}

(b) $a = 6(5) - 10 = 20$ m s^{-2}

(c) $v = 0$
$3t^2 - 10t + 3 = 0$
$(3t - 1)(t - 3) = 0$
$t = \frac{1}{3}$ or $t = 3$

(d) Maximum velocity, $\frac{dv}{dt} = 0$

$a = \frac{dv}{dt} = 6t - 10$
$6t - 10 = 0$
$t = \frac{5}{3}$
$t = \frac{5}{3}, v = 3\left(\frac{5}{3}\right)^2 - 10\left(\frac{5}{3}\right) + 3$
$= -5\frac{1}{3}$ m s^{-1}

Thus, the maximum velocity is $5\frac{1}{3}$ m s^{-1} to the left.

Self Test 3

1 $v = 4t^2 - 9$
Maximum displacement, $\frac{ds}{dt} = 0$
$4t^2 - 9 = 0$
$(2t + 3)(2t - 3) = 0$
$t = \frac{3}{2}$

$s = \int 4t^2 - 9\,dt$

$= \frac{4}{3}t^3 - 9t + c$

$t = 0, s = 0 \Rightarrow c = 0$

$\therefore s = \frac{4}{3}t^3 - 9t$

$t = \frac{3}{2}, s = \frac{4}{3}\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right)$

$= -9$ m

2 $v = t^2 - 5t + 4$

(a) Object stops instantaneously, $v = 0$

$t^2 - 5t + 4 = 0$

$(t - 4)(t - 1) = 0$

$t = 1, \quad t = 4$

$s = \int t^2 - 5t + 4\,dt$

$= \frac{t^3}{3} - \frac{5t^2}{2} + 4t + c$

$t = 0, s = 0 \Rightarrow c = 0$

$\therefore s = \frac{t^3}{3} - \frac{5t^2}{2} + 4t$

$t = 1, s = \frac{1}{3} - \frac{5}{2} + 4 = \frac{11}{6}$ m

(b) Minimum displacement, $\frac{ds}{dt} = 0$

$v = \frac{ds}{dt} = t^2 - 5t + 4$

$t^2 - 5t + 4 = 0$

$(t - 4)(t - 1) = 0$

$t = 4, t = 1$

$t = 1, s = \frac{11}{6}$ m

$t = 4, s = \frac{4^3}{3} - \frac{5}{2}(4^2) + 4(4)$

$= -\frac{8}{3}$ m

Thus, minimum displacement is $-\frac{8}{3}$ m

Minimum velocity, $\frac{dv}{dt} = 0$

$2t - 5 = 0$

$t = \frac{5}{2}$

$t = \frac{5}{2}, \quad v = \left(\frac{5}{2}\right)^2 - 5\left(\frac{5}{2}\right) + 4$

$= -\frac{9}{4}$ m s^{-1}

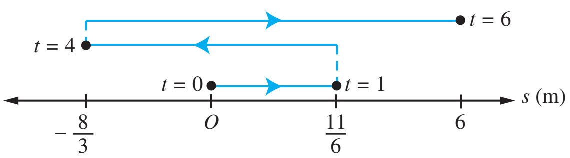
(c) $t = 0, s = 0$

$t = 1, s = \frac{11}{6}$ m

$t = 4, s = -\frac{8}{3}$ m

$t = 6, s = \frac{6^3}{3} - \frac{5}{2}(6^2) + 4(6)$

$= 6$ m



Total distance $= \frac{11}{6} + \frac{11}{6} + \frac{8}{3} + \frac{8}{3} + 6$

$= 15$ m

3 (a) $a = 18 - 3t$

Maximum velocity, $\frac{dv}{dt} = 0$

$18 - 3t = 0$

$t = 6$

$v = \int 18 - 3t\,dt$

$= 18t - \frac{3}{2}t^2 + c$

$t = 0, v = 0 \Rightarrow c = 0$

$\therefore v = 18t - \frac{3}{2}t^2$

$t = 6, v = 18(6) - \frac{3}{2}(6^2)$

$= 54$ m s^{-1}

(c) When object changes direction, $v = 0$

$18t - \frac{3}{2}t^2 = 0$

$36t - 3t^2 = 0$

$3t(12 - t) = 0$

$t = 0, t = 12$

The object changes direction at the 12th second.

(d) $s = \int 18t - \frac{3}{2}t^2\,dt$

$= 9t^2 - \frac{1}{2}t^3 + c$

$t = 0, s = 0 \Rightarrow c = 0$

$\therefore s = 9t^2 - \frac{1}{2}t^3$

When $v = 0, t = 12$

$t = 0, s = 9(12)^2 - \frac{1}{2}(12)^3$

$= 432$ m

$t = 15, s = 9(15)^2 - \frac{1}{2}(15)^3$

$= 337.5$ m

Total distance $= 432 + (432 - 337.5)$

$= 526.5$ m

(d) Maximum displacement, $\frac{ds}{dt} = 0$

$v = \frac{ds}{dt} = 0, t = 12,$

$s = 432$ m

Self Test 4

1 $h = 80 - 4t - 4t^2$

(a) Height of building, $t = 0, h = 80$ m

(b) $v = \frac{dh}{dt} = -4 - 8t$

$a = \frac{dv}{dt} = -8$ m s^{-2}

(c) Touches ground, $h = 0$

$-4t^2 - 4t + 80 = 0$

$t^2 + t - 20 = 0$

$(t + 5)(t - 4) = 0$

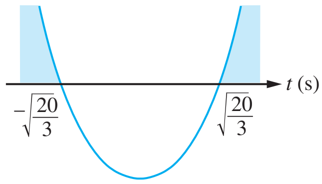
$t = 4$

$\therefore$ Velocity, $v = -4 - 8(4)$

$= -36$ m s^{-1}

2 $v = 6t^2 - 40$

(a)
$$v > 0$$
$$6t^2 - 40 > 0$$
$$2(3t^2 - 20) > 0$$
$$t > \sqrt{\frac{20}{3}}$$
$$t > 2\sqrt{\frac{5}{3}}$$
$$t > \frac{2\sqrt{15}}{3}$$



(b) Acceleration, $a = \frac{dv}{dt} = 12t$

(c)
$$a = 24$$
$$12t = 24$$
$$t = 2$$
$$v = 6(2^2) - 40$$
$$= 24 - 40$$
$$= -16 \text{ m s}^{-1}$$

SPM Practice

Paper 2

1 $s = 8t + t^2 - \frac{t^3}{3}$

(a) $v = \frac{ds}{dt} = 8 + 2t - t^2$

$t = 0, v = 8 \text{ m s}^{-1}$

$a = \frac{dv}{dt} = 2 - 2t$

$t = 0, a = 2 \text{ m s}^{-2}$

(b) Maximum velocity, $\frac{dv}{dt} = 0$

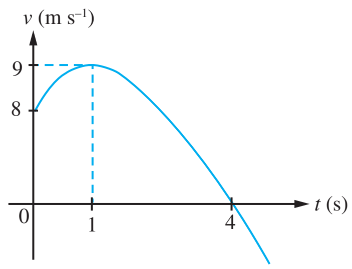
$2 - 2t = 0$

$t = 1$

$t = 1, v = 8 + 2(1) - 1^2$

$= 9 \text{ m s}^{-1}$

(c)



$$v = 0$$
$$8 + 2t - t^2 = 0$$
$$t^2 - 2t - 8 = 0$$
$$(t + 2)(t - 4) = 0$$
$$t = 4$$

The object changes direction at the 4th second.

(d) $t = 4, s = 8(4) + 4^2 - \frac{4^3}{3}$

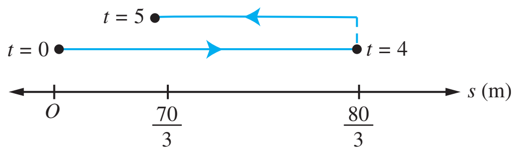
$= \frac{80}{3}$ m

(e) $t = 0, s = 0$

$t = 4, s = \frac{80}{3}$

$t = 5, s = 8(5) + 5^2 - \frac{5^3}{3}$

$= \frac{70}{3}$ m



Total distance $= \frac{80}{3} + \left(\frac{80}{3} - \frac{70}{3}\right) = 30$ m

2 $v = 2t^2 - 3t - 5$

(a) $t = 0, v = -5 \text{ m s}^{-1}$

$a = \frac{dv}{dt} = 4t - 3$

$t = 0, a = 4(0) - 3$

$= -3 \text{ m s}^{-2}$

(b) Minimum velocity, $\frac{dv}{dt} = 0$

$4t - 3 = 0$

$t = \frac{3}{4}$

$t = \frac{3}{4}, v = 2\left(\frac{3}{4}\right)^2 - 3\left(\frac{3}{4}\right) - 5$

$= -\frac{49}{8} \text{ m s}^{-1}$

(c) Object changes direction, $v = 0$

$2t^2 - 3t - 5 = 0$

$(t + 1)(2t - 5) = 0$

$t = \frac{5}{2}$

$t = \frac{5}{2}, a = 4\left(\frac{5}{2}\right) - 3$

$= 7 \text{ m s}^{-2}$

(d)
$$v = -5$$
$$2t^2 - 3t - 5 = -5$$
$$2t^2 - 3t = 0$$
$$t(2t - 3) = 0$$
$$t = 0, \quad t = 1.5$$
$$t = 1.5, \ a = 4(1.5) - 3$$
$$= 3 \text{ m s}^{-2}$$

3 $v = t^2 - 2t - 15$

(a)
$$v = 0$$
$$t^2 - 2t - 15 = 0$$
$$(t + 3)(t - 5) = 0$$
$$t = 5$$

(b) $s = \int t^2 - 2t - 15 \, dt$

$= \frac{t^3}{3} - t^2 - 15t + c$

$t = 0, s = 0 \Rightarrow c = 0$

$\therefore s = \frac{t^3}{3} - t^2 - 15t$

$t = 4, s = \frac{4^3}{3} - 4^2 - 15(4)$

$= -54.67 \text{ m s}^{-1}$

$t = 3, s = \frac{3^3}{3} - 3^2 - 15(3)$

$= -45 \text{ m s}^{-1}$

Distance travelled at the 4th second

$= |-54.67 - (-45)|$

$= 9.67$ m

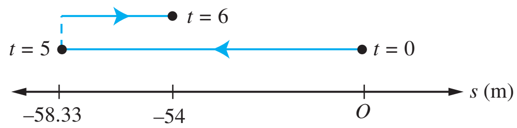
(c) $t = 5, v = 0$

$t = 0, s = 0$

$t = 5, s = \frac{5^3}{3} - 5^2 - 15(5)$

$= -58.33$ m

$t = 6,\ s = \frac{6^3}{3} - 6^2 - 15(6)$
$= -54$



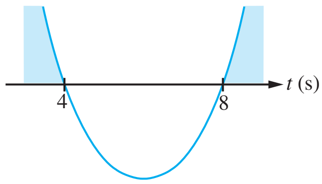
Total distance $= 58.33 + (58.33 - 54)$
$= 62.66$ m

(d) Maximum displacement, $\frac{ds}{dt} = 0$
$t = 5, v = 0$
Thus, maximum displacement achieved
$= -58.33$ m

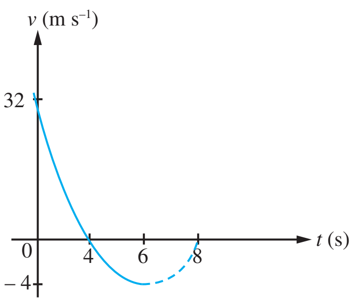
4 (a) $v = t^2 - 12t + k$
Initial velocity, $t = 0$, $v = 32$ m s^{-1}

(i) $t = 0, v = 32$
$\therefore k = 32$

(ii) $v > 0$
$t^2 - 12t + 32 > 0$
$(t - 4)(t - 8) > 0$
$\therefore 0 < t < 4, t > 8$



(b) $t = 6,\ v = 6^2 - 12(6) + 32$
$= -4$ m s^{-1}



Total distance $= \int_0^4 t^2 - 12t + 32\, dt + \left|\int_4^6 t^2 - 12t + 32\, dt\right|$

$= \left[\frac{t^3}{3} - 6t^2 + 32t\right]_0^4 + \left|\left[\frac{t^3}{3} - 6t^2 + 32t\right]_4^6\right|$

$= \left[\frac{4^3}{3} - 6(4^2) + 32(4)\right] + \left|\left[\frac{6^3}{3} - 6(6^2) + 32(6)\right] - \left[\frac{4^3}{3} - 6(4^2) + 32(4)\right]\right|$

$= 53.33 + |48 - 53.33|$
$= 58.66$ m

5 $s = \frac{2}{3}t^3 - 4t^2 + 6t$

$s = 0$
$\frac{2}{3}t^3 - 4t^2 + 6t = 0$
$2t^3 - 12t^2 + 18t = 0$
$2t(t^2 - 6t + 9) = 0$
$2t(t - 3)^2 = 0$
$t = 3$
$T = 3$

(a) Maximum displacement, $\frac{ds}{dt} = 0$
$2t^2 - 8t + 6 = 0$
$t^2 - 4t + 3 = 0$
$(t - 1)(t - 3) = 0$
$t = 1, t = 3$

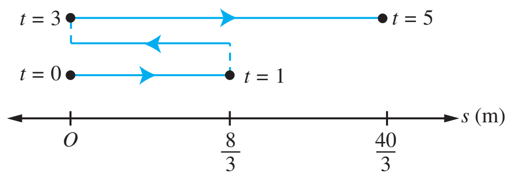
$t = 1, s = \frac{2}{3} - 4 + 6 = \frac{8}{3}$ m

$t = 3, s = \frac{2}{3}(3^3) - 4(3^2) + 6(3)$
$= 0$

Thus, maximum displacement is $\frac{8}{3}$ m.

(b) Acceleration, $a = \frac{d^2s}{dt^2} = 4t - 8$
$t = 3, a = 4(3) - 8 = 4$ m s^{-2}

(c) $t = 1, s = \frac{8}{3}$
$t = 3, s = 0$
$t = 5, s = \frac{2}{3}(5^3) - 4(5^2) + 6(5)$
$= \frac{40}{3}$ m



Total distance $= \frac{8}{3}(2) + \frac{40}{3}$
$= \frac{56}{3}$ m

6 (a) $t = 0, v = 10$
$v = pt^2 + 2t + q$
$t = 0, q = 10,$
$\therefore v = pt^2 + 2t + 10$
$a = \frac{dv}{dt} = 2pt + 2$
$t = 2, a = 4,$
$4 = 2p(2) + 2$
$4p = 2$
$p = \frac{1}{2}$

(b) $a = t + 2$
$t = 4, a = 4 + 2 = 6$ m s^{-2}

(c) $v = \frac{1}{2}t^2 + 2t + 10$

$s = \int \frac{1}{2}t^2 + 2t + 10\, dt$

$= \frac{1}{6}t^3 + t^2 + 10t + c$

$t = 0, s = 0 \Rightarrow c = 0$

$\therefore s = \frac{1}{6}t^3 + t^2 + 10t$

$t = 4, s = \frac{1}{6}(4^3) + 4^2 + 10(4)$
$= 66\frac{2}{3}$ m

$t = 3, s = \frac{1}{6}(3^3) + 3^2 + 10(3)$
$= 43\frac{1}{2}$ m

Displacement at $t = 4$ seconds,

$= \left|66\frac{2}{3} - 43\frac{1}{2}\right|$

$= 23\frac{1}{6}$ m

7 $v_P = 4 - 5t + t^2$

(a) Maximum velocity, $\frac{dv_P}{dt} = 0$

$-5 + 2t = 0$

$t = 2.5$

$t = 2.5,\ v = 4 - 5(2.5) + (2.5)^2$

$= -2.25$ m s^{-1}

(b) $v_P = 0$

$4 - 5t + t^2 = 0$

$(t - 1)(t - 4) = 0$

$t = 1, \quad t = 4$

Thus, object P stops instantaneously for the first time at $t = 1$ second.
Assume A as the starting point with $s = 0$.

$s_P = \int 4 - 5t + t^2\, dt$

$= 4t - \frac{5}{2}t^2 + \frac{1}{3}t^3 + c$

$t = 0, s = 0 \Rightarrow c = 0$

$\therefore s_P = 4t - \frac{5}{2}t^2 + \frac{1}{3}t^3$

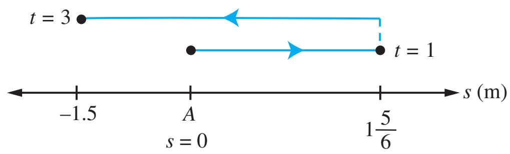
$t = 1,\ s_P = 4 - \frac{5}{2} + \frac{1}{3}$

$= 1\frac{5}{6}$ m

Thus, distance of AC is $1\frac{5}{6}$ m.

(c) $t = 3,\ s = 4(3) - \frac{5}{2}(3^2) + \frac{3^3}{3}$

$= -1.5$ m



Distance travelled by object $Q = 5(3)$

$= 15$ m (arrived at point A)

Thus, the distance between P and Q at $t = 3$ seconds is 1.5 m.