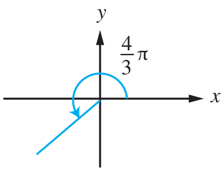
FORM 5

CHAPTER 6 Trigonometric Functions

Self Test 1

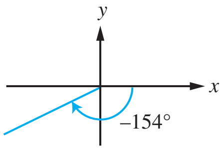
1 (a)



Quadrant III

$\frac{4}{3}\pi = \frac{4}{3}(180°)$
$= 240°$

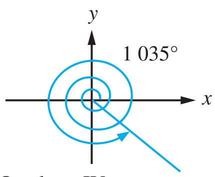
(b)



Quadrant III

$-154° = 360° - 154°$
$= 206°$

(c)



Quadrant IV

$1\,035° = 1\,035° - 720°$
$= 315°$

Self Test 2

1 (a) $\cos 57° = \cos(90° - 33°)$
$= \sin 33°$
$= b$

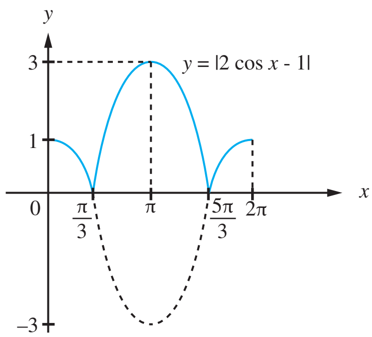
(b) $\text{cosec } 147° = \frac{1}{\sin 147°}$
$= \frac{1}{\sin(180° - 33°)}$
$= \frac{1}{b}$

(c) $\sec 213° = \frac{1}{\cos 213°}$
$= \frac{1}{\cos(180° + 33°)}$
$= -\frac{1}{\cos 33°}$
$= -\frac{1}{a}$

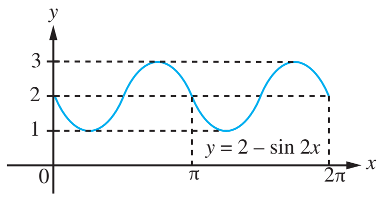
(d) $\frac{11}{60}\pi = \frac{11}{60}(180°) = 33°$
$\sin \frac{11}{60}\pi = \sin 33°$
$= b$

Self Test 3

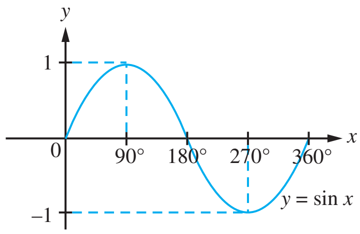
1 (a)



(b) $y = 2 - \sin 2x = -\sin 2x + 2$



2 (a)



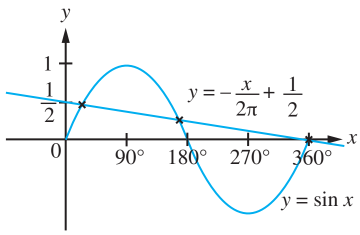
(b) $\frac{x}{2\pi} + 2\sin x - 1 = 0$

$2\sin x = 1 - \frac{x}{2\pi}$

$\sin x = -\frac{x}{4\pi} + \frac{1}{2}$

$\therefore y = -\frac{x}{4\pi} + \frac{1}{2}$

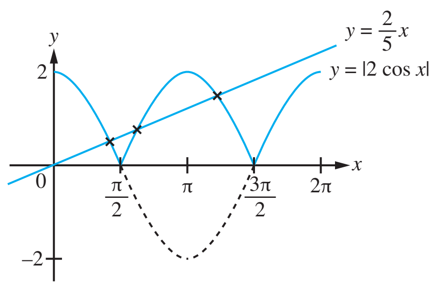
(c)



Number of solutions = 3

© EPH Publishing (M) Sdn. Bhd. (199801017497) 2024

3



Number of solutions = 3

Self Test 4

1 (a) $\tan x + \cot x = \sec x \operatorname{cosec} x$

Left-hand side:

$$\tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$
$$= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$
$$= \frac{1}{\sin x \cos x}$$
$$= \left(\frac{1}{\sin x}\right)\left(\frac{1}{\cos x}\right)$$
$= \sec x \operatorname{cosec} x$ (Right-hand side)

(b) $\frac{1 - \sin x}{\cos x} + \frac{\cos x}{1 - \sin x} = 2 \sec x$

Left-hand side:

$$\frac{1 - \sin x}{\cos x} + \frac{\cos x}{1 - \sin x} = \frac{(1 - \sin x)^2 + \cos^2 x}{\cos x\,(1 - \sin x)}$$
$$= \frac{1 - 2\sin x + \sin^2 x + \cos^2 x}{\cos x(1 - \sin x)}$$
$$= \frac{2 - 2\sin x}{\cos x(1 - \sin x)}$$
$$= \frac{2(1 - \sin x)}{\cos x(1 - \sin x)}$$
$$= \frac{2}{\cos x}$$
$= 2 \sec x$ (Right-hand side)

2 $\cot^2 x - \cos^2 x = \cot^2 x \cos^2 x$

Left-hand side: $\cot^2 x - \cos^2 x = \frac{\cos^2 x}{\sin^2 x} - \cos^2 x$
$$= \frac{\cos^2 x - \sin^2 x \cos^2 x}{\sin^2 x}$$
$$= \frac{\cos^2 x(1 - \sin^2 x)}{\sin^2 x}$$
$$= \frac{\cos^2 x}{\sin^2 x}(\cos^2 x)$$
$= \cot^2 x \cos^2 x$ (Right-hand side)

Self Test 5

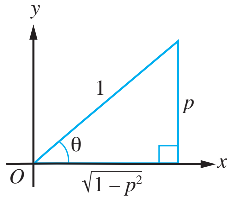
1 (a) $\frac{\cos^2 A}{1 + \sin A} + \sin A = 1$

$$\frac{\cos^2 A}{1 + \sin A} + \sin A = \frac{\cos^2 A + \sin A + \sin^2 A}{1 + \sin A}$$
$$= \frac{1 + \sin A}{1 + \sin A}$$
$= 1$ (Proven)

(b) $\frac{\sin A + \tan A}{1 + \cos A} = \tan A$

$$\frac{\sin A + \tan A}{1 + \cos A} = \frac{\sin A + \frac{\sin A}{\cos A}}{1 + \cos A}$$
$$= \frac{\sin A \cos A + \sin A}{\cos A(1 + \cos A)}$$
$$= \frac{\sin A(\cos A + 1)}{\cos A(1 + \cos A)}$$
$= \tan A$ (Proven)

2 $\sin \theta = p$



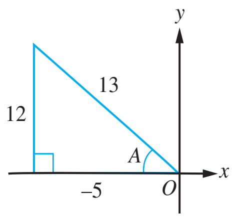
(a) $\tan \theta = \frac{p}{\sqrt{1 - p^2}}$

(b) $$\cot(\pi - \theta) = \frac{1}{\tan(\pi - \theta)}$$
$$= -\frac{1}{\tan \theta}$$
$$= -\frac{\sqrt{1 - p^2}}{p}$$

(c) $\sin(0.5\pi + \theta) = \sin(0.5\pi)\cos\theta + \cos(0.5\pi)\sin\theta$
$= \cos\theta + 0$
$= \sqrt{1 - p^2}$

(d) $$\operatorname{cosec}(2\pi - \theta) = \frac{1}{\sin(2\pi - \theta)}$$
$$= -\frac{1}{\sin\theta}$$
$$= -\frac{1}{p}$$

3



(a) $\cos 2A = 2\cos^2 A - 1$
$$= 2\left(-\frac{5}{13}\right)^2 - 1$$
$$= 2\left(\frac{25}{169}\right) - 1$$
$$= -\frac{119}{169}$$

(b) $\cos A = 1 - 2\sin^2 \frac{A}{2}$

$$2\sin^2 \frac{A}{2} = 1 - \cos A$$
$$\sin^2 \frac{A}{2} = \frac{1 - \cos A}{2}$$
$$\sin \frac{A}{2} = \sqrt{\frac{1 - \cos A}{2}}$$
$$= \sqrt{\frac{1 - \left(-\frac{5}{13}\right)}{2}}$$
$$= \sqrt{\frac{18}{26}}$$
$$= \sqrt{\frac{9}{13}}$$

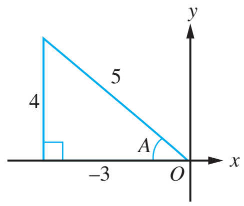
(c) $\sin 3A = \sin(A + 2A)$
$= \sin A \cos 2A - \cos A \sin 2A$

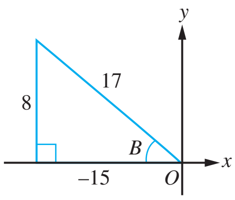
$\cos 2A = -\frac{119}{169}$ ← Answer in (a)

© EPH Publishing (M) Sdn. Bhd. (199801017497) 2024

$$\sin 2A = 2\sin A\cos A$$
$$= 2\left(\frac{12}{13}\right)\left(-\frac{5}{13}\right)$$
$$= -\frac{120}{169}$$
$$\sin 3A = \left(\frac{12}{13}\right)\left(-\frac{119}{169}\right) - \left(-\frac{5}{13}\right)\left(-\frac{120}{169}\right)$$
$$= \frac{-1\ 428 - 600}{2\ 197}$$
$$= -\frac{12}{13}$$

4





(a) $\sin(A + B) = \sin A\cos B + \cos A\sin B$
$$= \frac{4}{5}\left(-\frac{15}{17}\right) + \left(-\frac{3}{5}\right)\left(\frac{8}{17}\right)$$
$$= -\frac{84}{85}$$

(b) $\tan(360° - B) = -\tan B$
$$= -\left(-\frac{8}{15}\right)$$
$$= \frac{8}{15}$$

(c) $\cos\frac{B}{2} - \cos(A + 45°)$
$$\cos B = 2\cos^2\frac{B}{2} - 1$$
$$-\frac{15}{17} = 2\cos^2\frac{B}{2} - 1$$
$$\frac{2}{17} = 2\cos^2\frac{B}{2}$$
$$\cos\frac{B}{2} = \sqrt{\frac{1}{17}}$$
$$\sqrt{\frac{1}{17}} - [\cos A\cos 45° - \sin A\sin 45°]$$
$$= \sqrt{\frac{1}{17}} - \left[\left(-\frac{3}{5}\right)\left(\frac{\sqrt{2}}{2}\right) - \frac{4}{5}\left(\frac{\sqrt{2}}{2}\right)\right]$$
$$= \sqrt{\frac{1}{17}} - \left(\frac{-7\sqrt{2}}{10}\right)$$
$$= \frac{\sqrt{17}}{17} + \frac{7\sqrt{2}}{10}$$

Self Test 6

1 (a) $2\sin x = \cos(x + 60°)$
$$2\sin x = \cos x\cos 60° - \sin x\sin 60°$$
$$2\sin x = \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$$
$$\left(2 + \frac{\sqrt{3}}{2}\right)\sin x = \frac{1}{2}\cos x$$
$$\frac{\sin x}{\cos x} = 0.1745$$
$$\tan x = 0.1745$$
$$x = 9.9°, 189.9°$$

(b) $3\sec^2 x = 5(1 + \tan x)$
$$3(1 + \tan^2 x) = 5 + 5\tan x$$
$$3 + 3\tan^2 x = 5 + 5\tan x$$
$$3\tan^2 x - 5\tan x - 2 = 0$$
$$(3\tan x + 1)(\tan x - 2) = 0$$
$$\tan x = -\frac{1}{3}, \quad \tan x = 2$$
$$x = 63.4°, 161.57°, 243.4°, 341.57°$$

2 (a) $5\cos x + 3\sin x = 0$
$$3\sin x = -5\cos x$$
$$\frac{\sin x}{\cos x} = -\frac{5}{3}$$
$$\tan x = -\frac{5}{3}$$
(Quadrants II and IV)
$$x = 120.96°, 300.96°$$

(b) $2\cos^2 x - 3\cos x + 1 = 0$
$$(2\cos x - 1)(\cos x - 1) = 0$$
$$\cos x = \frac{1}{2}, \cos x = 1$$
$$x = 0°, 60°, 300°, 360°$$

(c) $\sec 2x = -2.5896$
$$\frac{1}{\cos 2x} = -2.5896$$
$$\cos 2x = -0.3862$$
(Quadrants II and III)
$$2x = 112.72°, 247.28°, 472.72°, 607.28°$$
$$x = 56.36°, 123.64°, 236.36°, 303.64°$$

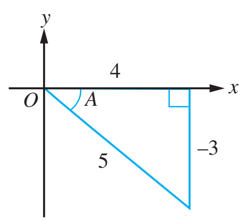
3 (a) $2\operatorname{cosec}^2 x + \cot x - 8 = 0$
$$2(\cot^2 x + 1) + \cot x - 8 = 0$$
$$2\cot^2 x + \cot x - 6 = 0$$
$$(2\cot x - 3)(\cot x + 2) = 0$$
$$\cot x = \frac{3}{2}, \qquad \cot x = -2$$
$$\tan x = \frac{2}{3} \qquad \tan x = -\frac{1}{2}$$
$$x = 33.69°, 153.43°, 213.69°, 333.43°$$

(b) $2\cos 2x + 3\sin x - 2 = 0$
$$2(1 - 2\sin^2 x) + 3\sin x - 2 = 0$$
$$2 - 4\sin^2 x + 3\sin x - 2 = 0$$
$$4\sin^2 x - 3\sin x = 0$$
$$\sin x(4\sin x - 3) = 0$$
$$\sin x = 0, \sin x = \frac{3}{4}$$
$$x = 0°, 48.6°, 131.4°, 180°, 360°$$

SPM Practice

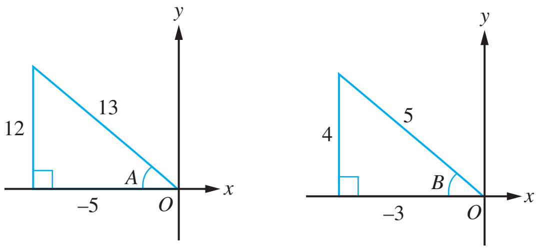
Paper 1

1 (a)



$$\sin 2A = 2\sin A\cos A$$
$$= 2\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right)$$
$$= -\frac{24}{25}$$

(b)



$\tan(A+B)\tan(A-B)$

$= \dfrac{\tan A + \tan B}{1 - \tan A \tan B} \times \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

$= \dfrac{\tan^2 A - \tan^2 B}{1 - (\tan^2 A \tan^2 B)}$

$= \dfrac{\left(-\frac{12}{5}\right)^2 - \left(-\frac{4}{3}\right)^2}{1 - \left(-\frac{12}{5}\right)^2\left(-\frac{4}{3}\right)^2}$

$= \dfrac{\frac{144}{25} - \frac{16}{9}}{1 - \frac{256}{25}}$

$= -\dfrac{128}{297}$

2 (a) $\tan 3x = -2$

$3x = 116.57°, 296.57°, 476.57°, 656.57°, 836.57°, 1\,016.57°$

$x = 38.86°, 98.86°, 158.86°, 218.86°, 278.86°, 338.86°$

(b) $\dfrac{1}{\sin\left(\frac{x}{2}\right)} = 1.542$

$\sin\left(\frac{x}{2}\right) = 0.6485$

$\dfrac{x}{2} = 40.43°, 139.57°$

$x = 80.86°, 279.14°$

3 $(3\sin A + \cos A)(\sin A - \cos A)$

$= 3\sin^2 A - 3\sin A\cos A + \sin A\cos A - \cos^2 A$

$= 3\sin^2 A - 2\sin A\cos A - \cos^2 A$

$= 3\sin^2 A - (1 - \sin^2 A) - 2\sin A\cos A$

$= 4\sin^2 A - 2\sin A\cos A - 1$

$= 4\sin^2 A - 2\sin 2A - 1$

Thus,

$4\sin^2 A - 2\sin A\cos A - 1 = 0$

$(3\sin A + \cos A)(\sin A - \cos A) = 0$

$3\sin A = -\cos A, \quad \sin A = \cos A$

$\tan A = -\dfrac{1}{3} \qquad \tan A = 1$

$A = 45°, 161.57°, 225°, 341.57°$

4 $\tan A = 3, \tan(A+B) = \dfrac{1}{7}$

(a) $\dfrac{\tan A + \tan B}{1 - \tan A \tan B} = \dfrac{1}{7}$

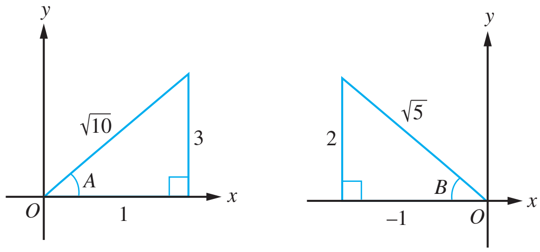
$\dfrac{3 + \tan B}{1 - 3\tan B} = \dfrac{1}{7}$

$21 + 7\tan B = 1 - 3\tan B$

$10\tan B = -20$

$\tan B = -2$

(b)



$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$= \dfrac{1}{\sqrt{10}}\left(-\dfrac{1}{\sqrt{5}}\right) - \left(\dfrac{3}{\sqrt{10}}\right)\left(\dfrac{2}{\sqrt{5}}\right)$

$= -\dfrac{1}{\sqrt{50}} - \dfrac{6}{\sqrt{50}}$

$= -\dfrac{7}{\sqrt{50}}$

$= -\dfrac{7}{5\sqrt{2}}$

$= -\dfrac{7\sqrt{2}}{10}$

5 (a) $\sin\left(2x - \dfrac{\pi}{6}\right) = \dfrac{1}{2}$

$2x - \dfrac{\pi}{6} = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{13\pi}{6}, \dfrac{17\pi}{6}$

$2x = \dfrac{\pi}{3}, \pi, \dfrac{7\pi}{3}, 3\pi$

$x = \dfrac{\pi}{6}, \dfrac{\pi}{2}, \dfrac{7\pi}{6}, \dfrac{3\pi}{2}$

(b) $\dfrac{\tan^2 x - 1}{\sin x + \cos x}$

$= \dfrac{\frac{\sin^2 x}{\cos^2 x} - 1}{\sin x + \cos x}$

$= \dfrac{(\sin^2 x - \cos^2 x)}{\cos^2 x(\sin x + \cos x)}$

$= \dfrac{(\sin x - \cos x)(\sin x + \cos x)}{\cos^2 x(\sin x + \cos x)}$

$= \dfrac{\sin x - \cos x}{\cos^2 x}$

6 (a) $1 - 2\sin^2 x = 1 + \sin x$

$2\sin^2 x + \sin x = 0$

$\sin x(2\sin x + 1) = 0$

$\sin x = 0, \ \sin x = -\dfrac{1}{2}$

$x = 0°, 180°, 210°, 330°, 360°$

(b) $\cot^2 x + 8 = 7\operatorname{cosec} x$

$\operatorname{cosec}^2 x - 1 + 8 = 7\operatorname{cosec} x$

$\operatorname{cosec}^2 x - 7\operatorname{cosec} x + 7 = 0$

$\operatorname{cosec} x = 5.7913, \quad \operatorname{cosec} x = 1.209$

$\dfrac{1}{\sin x} = 5.7913 \qquad \dfrac{1}{\sin x} = 1.209$

$\sin x = 0.1727 \qquad \sin x = 0.8271$

$x = 9.94°, 55.8°, 124.2°, 170.06°$

© EPH Publishing (M) Sdn. Bhd. (199801017497) 2024

Paper 2

1 (a) (i) $7 - 5\sin x = 6(1 - \sin^2 x)$

$7 - 5\sin x = 6 - 6\sin^2 x$

$6\sin^2 x - 5\sin x + 1 = 0$

$(3\sin x - 1)(2\sin x - 1) = 0$

$\sin x = \frac{1}{3}, \quad \sin x = \frac{1}{2}$

$x = 19.5°, 30°, 150°, 160.5°$

(ii) $2(2\sin 2x - 3\cos x) = 0$

$2\sin 2x - 3\cos x = 0$

$4\sin x\cos x - 3\cos x = 0$

$\cos x(4\sin x - 3) = 0$

$\cos x = 0, \quad \sin x = \frac{3}{4}$

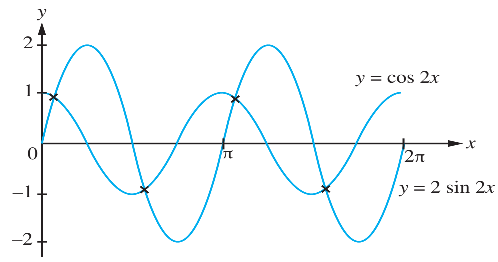
$x = 48.6°, 90°, 131.4°, 270°$

(b) $\tan 2x = 0.5$

$\frac{\sin 2x}{\cos 2x} = \frac{1}{2}$

$2\sin 2x = \cos 2x$

$y = \cos 2x$



Number of solutions = 4

2 (a) $(1 + \cot\theta)^2 + (1 - \cot\theta)^2 = \frac{2}{\sin^2\theta}$

Left-hand side:

$\left(1 + \frac{\cos\theta}{\sin\theta}\right)^2 + \left(1 - \frac{\cos\theta}{\sin\theta}\right)^2$

$= \left(\frac{\sin\theta + \cos\theta}{\sin\theta}\right)^2 + \left(\frac{\sin\theta - \cos\theta}{\sin\theta}\right)^2$

$= \frac{\sin^2\theta + \cos^2\theta + 2\cos\theta\sin\theta}{\sin^2\theta} + \frac{\sin^2\theta + \cos^2\theta - 2\cos\sin\theta}{\sin^2\theta}$

$= \frac{1 + 2\sin\theta\cos\theta - 2\sin\theta\cos\theta + 1}{\sin^2\theta}$

$= \frac{2}{\sin^2\theta}$ (Right-hand side)

(b) $y = 4\sin x\cos x$

$= 2(\sin x\cos x)$

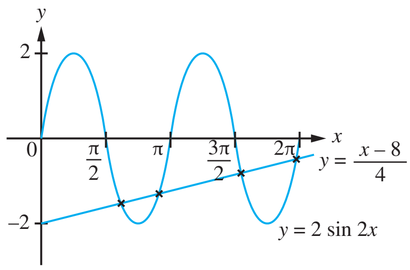
$= 2\sin 2x$

$\frac{\sin 2x}{2} = \frac{x - 8}{16}$

$\frac{4\sin 2x}{2} = \frac{4(x - 8)}{16}$

$2\sin 2x = \frac{(x - 8)}{4}$

$y = \frac{(x - 8)}{4}$



Number of solutions = 4

© EPH Publishing (M) Sdn. Bhd. (199801017497) 2024