FORM 5

CHAPTER 5 Probability Distribution

Self Test 1

1 (a) Discrete random variable

(b) $X = \{0, 1, 2, 3\}$

(c) $P(X = 0) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

$P(X = 1) = 3 \times \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) = \frac{3}{8}$

$P(X = 2) = 3 \times \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) = \frac{3}{8}$

$P(X = 3) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

$X = x$	0	1	2	3
$P(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

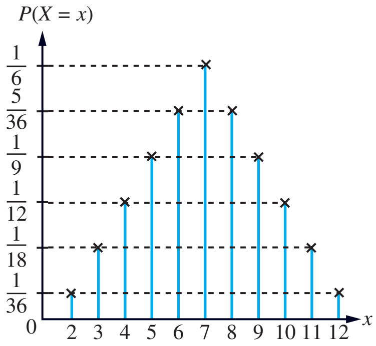
2 (a) Discrete random variable because the scores obtained in an archery game can only take the values on the targets.
(b) Continuous random variable because the evaluation marks for students' skills can have any value.
(c) Continuous random variable because the mass of eggs can take any value.
(d) Discrete random variable because the number of cars is countable.
(e) Continuous random variable because the time taken is in hours, minutes and seconds.

3 (a) $X = \{x : x = \text{total numbers obtained from both dice}\}$
$X = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

(b)

$X = x$	$P(X = x)$
2	$\frac{1}{36}$
3	$\frac{2}{36} = \frac{1}{18}$
4	$\frac{3}{36} = \frac{1}{12}$
5	$\frac{4}{36} = \frac{1}{9}$
6	$\frac{5}{36}$
7	$\frac{6}{36} = \frac{1}{6}$
8	$\frac{5}{36}$
9	$\frac{4}{36} = \frac{1}{9}$
10	$\frac{3}{36} = \frac{1}{12}$
11	$\frac{2}{36} = \frac{1}{18}$
12	$\frac{1}{36}$

(c) Probability distribution graph for X:



4 (a) Discrete random variable because x can take any integer values such as 2, 3, 4, 5 and 6.

(b) $k + 0.05 + 2k + 0.3 + k + 0.2 = 1$
$4k + 0.55 = 1$
$4k = 0.45$
$k = 0.1125$

(c) $P(X = \text{odd number}) = P(X = 3) + P(X = 5)$
$= 0.05 + k$
$= 0.05 + 0.1125$
$= 0.1625$

Self Test 2

1 Standard deviation $= 7$, Variance $= npq = 49$
$P(\text{bread sold}) = 0.98$
$q = 1 - 0.98 = 0.02$
$\therefore n(0.98)(0.02) = 49$
$n = 2\,500$

2 (a) $P(\text{female teachers}) = 0.8$
$n = 20$, X = number of female teachers

(i) $P(X = 10) = {}^{20}C_{10}\,(0.8)^{10}\,(0.2)^{10}$
$= 0.00203$

(ii) 25% of female teachers $= \frac{25}{100} \times 20$
$= 5$
$P(X = 5) = {}^{20}C_5(0.8)^5(0.2)^{15}$
$= 1.665 \times 10^{-7}$

(b) Mean $= np$
$36 = n(0.8)$
$n = 45$

3 (a) $p = 0.4$, $n = 5$
$P(X \geqslant 3) = P(X = 3) + P(X = 4) + P(X = 5)$
$= {}^5C_3(0.4)^3(0.6)^2 + {}^5C_4(0.4)^4(0.6)^1 + {}^5C_5(0.4)^5(0.6)^0$
$= 0.2304 + 0.0768 + 0.01024$
$= 0.3174$

(b) $n = 7$, $p = 0.4$
Mean $= np$
$= 7(0.4)$
$= 2.8$
≈ 3
Variance $= npq$
$= 7(0.4)(0.6)$
$= 1.68$

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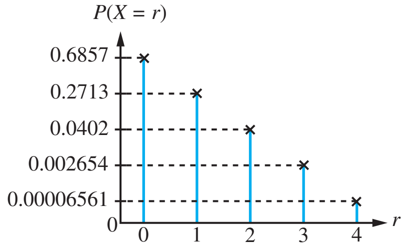
4 $n = 4, p = 0.09$

(a) X = number of donors with blood group B
$P(X > 2) = P(X = 3) + P(X = 4)$
$= {}^4C_3(0.09)^3(0.91)^1 + {}^4C_4(0.09)^4(0.91)^0$
$= 0.002654 + 0.00006561$
$= 0.00272$

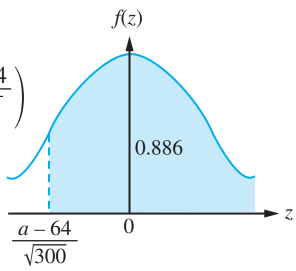
(b) $P(X = 0) = {}^4C_0(0.09)^0(0.91)^4 = 0.6857$
$P(X = 1) = {}^4C_1(0.09)^1(0.91)^3 = 0.2713$
$P(X = 2) = {}^4C_2(0.09)^2(0.91)^2 = 0.0402$
$P(X = 3) = 0.002654$
$P(X = 4) = 0.00006561$



(c) $n = 2\,000$
Mean $= np$
$= 2\,000(0.09)$
$= 180$
Variance $= npq$
$= 2\,000(0.09)(0.91)$
$= 163.8$
Standard deviation $= \sqrt{\text{Variance}}$
$= \sqrt{163.8}$
$= 12.8$

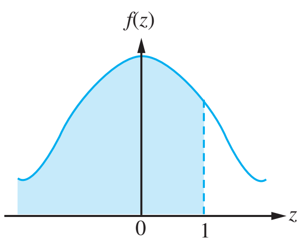
Self Test 3

1 $\mu = 64, \sigma = \sqrt{300} = 17.32$
$P(X > a) = 0.886$
$P\left(Z > \frac{a - 64}{\sqrt{300}}\right) = 0.886$
$P\left(Z < \frac{a - 64}{\sqrt{300}}\right) = P\left(Z > \frac{a - 64}{\sqrt{300}}\right)$
$= 1 - 0.886$
$= 0.114$
$\frac{64 - a}{\sqrt{300}} = 1.206$
$64 - a = 20.889$
$a = 43.11$

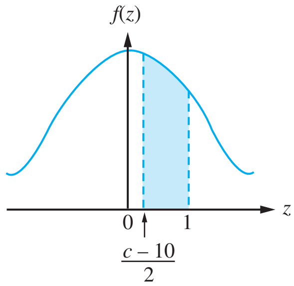


2 $\mu = 10, \sigma = \sqrt{4} = 2$

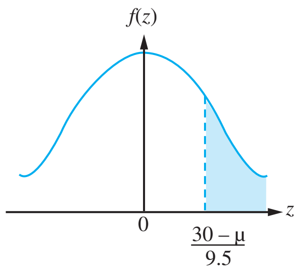
(a) $P(X < 12) = P\left(Z < \frac{12 - 10}{2}\right)$
$= P(Z < 1)$
$= 1 - P(Z > 1)$
$= 1 - 0.1587$
$= 0.8413$



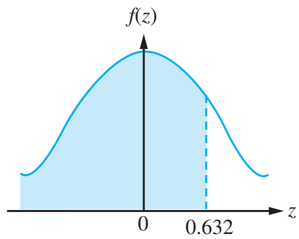
(b) $P(c < X < 12) = 0.12$
$P\left(\frac{c - 10}{2} < Z < \frac{12 - 10}{2}\right) = 0.12$
$P\left(\frac{c - 10}{2} < Z < 1\right) = 0.12$
$P\left(Z > \frac{c - 10}{2}\right) - P(Z > 1) = 0.12$
$P\left(Z > \frac{c - 10}{2}\right) = 0.12 + 0.1587$
$= 0.2787$
$\frac{c - 10}{2} = 0.587$
$c = 11.174$



3 (a) $P(X \geqslant 30) = 0.2$
$P\left(Z \geqslant \frac{30 - \mu}{9.5}\right) = 0.2$
$\frac{30 - \mu}{9.5} = 0.842$
$30 - \mu = 7.999$
$\mu = 22$
$\therefore$ Average = RM22



(b) $P(X < 28) = P\left(Z < \frac{28 - 22}{9.5}\right)$
$= P(Z < 0.632)$
$= 1 - P(Z > 0.632)$
$= 1 - 0.2636$
$= 0.7364$



4 (a) $k = 0, r = 1$ because the graph shows standard normal distribution $N(0, 1)$

(b) Area of shaded region $= P(0 < Z < 2)$
$= 0.5 - P(Z > 2)$
$= 0.5 - 0.0228$
$= 0.4772$

SPM Practice

Paper 1

1 $n = 20, P(\text{pass}) = \frac{2}{3}$

(a) $\frac{75}{100} \times 20 = 15$
$P(X = 15) = {}^{20}C_{15}\left(\frac{2}{3}\right)^{15}\left(\frac{1}{3}\right)^5$
$= 0.1457$

(b) $P(X = 10) = {}^{20}C_{10}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^{10}$
$= 0.0543$

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2 $p = \frac{2}{7}, n = 10$

(a) $P(X \leqslant 1) = P(X = 0) + P(X = 1)$
$= {}^{10}C_0\left(\frac{2}{7}\right)^0\left(\frac{5}{7}\right)^{10} + {}^{10}C_1\left(\frac{2}{7}\right)^1\left(\frac{5}{7}\right)^9$
$= 0.03457 + 0.1383$
$= 0.1729$

(b) P(more than one girl)
$P(X > 1) = 1 - P(X \leqslant 1)$
$= 1 - 0.1729$
$= 0.8271$
Number of groups $= 0.8271(100)$
$= 82.71$
≈ 82

3 (a) $n = 10, P(\text{correct}) = \frac{1}{4}$
P(All answers are wrong) $= P(X = 0)$
$= {}^{10}C_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^{10}$
$= 0.0563$

(b) $P(X > 7) = P(X = 8) + P(X = 9) + P(X = 10)$
$= {}^{10}C_8\left(\frac{1}{4}\right)^8\left(\frac{3}{4}\right)^2 + {}^{10}C_9\left(\frac{1}{4}\right)^9\left(\frac{3}{4}\right)^1 + {}^{10}C_{10}\left(\frac{1}{4}\right)^{10}\left(\frac{3}{4}\right)^0$
$= 0.000386 + 2.861 \times 10^{-5} + 9.54 \times 10^{-7}$
$= 0.000416$

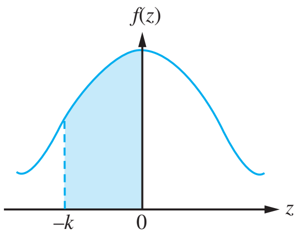
4 P(like to eat burger) $= 0.35, n = 9$

(a) (i) $P(X = 5) = {}^9C_5(0.35)^5(0.65)^4$
$= 0.1181$

(ii) $P(X \leqslant 7) = 1 - P(X = 8) - P(X = 9)$
$= 1 - {}^9C_8(0.35)^8(0.65)^1 - {}^9C_9(0.35)^9(0.65)^0$
$= 1 - 0.001317 - 7.882 \times 10^{-5}$
$= 0.9986$

(b) Mean $= np$
$= 520(0.65)$
$= 338$
Variance $= npq$
$= 520(0.65)(0.35)$
$= 118.3$
Standard deviation $= \sqrt{118.3} = 10.88$

5 $P(-k \leqslant Z \leqslant 0) = 0.381$
$0.5 - P(Z < -k) = 0.381$
$0.5 - P(Z > k) = 0.381$
$P(Z > k) = 0.119$
$k = 1.18$



6 $\mu = 250, \sigma = \sqrt{100} = 10$

(a) $P(X < 240) = P\left(Z < \frac{240 - 250}{10}\right)$
$= P(Z < -1)$
$= P(Z > 1)$
$= 0.1587$

(b) $P(X > 262) = 0.25$
$P\left(Z > \frac{262 - \mu}{10}\right) = 0.25$
$\frac{262 - \mu}{10} = 0.674$
$\mu = 255.26$ g
$\therefore r = 255.26$

Paper 2

1 (a) Discrete random variable because the number of seats must be an integer value.

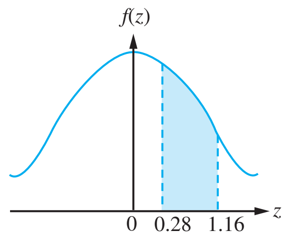
(b) $n = 7, p = 0.42$
Mean $= np$
$= 7(0.42)$
$= 2.94$
≈ 3
Variance $= npq$
$= 7(0.42)(0.58)$
$= 1.705$

(c) (i) $P(X = 0) = {}^7C_0(0.42)^0(0.58)^7$
$= 0.0221$

(ii) $P(X \leqslant 3)$
$= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$
$= 0.0221 + {}^7C_1(0.42)^1(0.58)^6 + {}^7C_2(0.42)^2(0.58)^5 + {}^7C_3(0.42)^3(0.58)^4$
$= 0.0221 + 0.1119 + 0.2431 + 0.2934$
$= 0.6705$

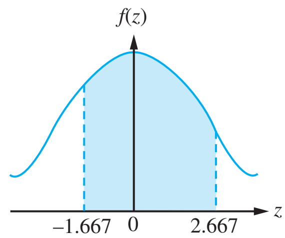
(iii) $P(3 \leqslant X < 5) = P(X = 3) + P(X = 4)$
$= 0.2934 + {}^7C_4(0.42)^4(0.58)^3$
$= 0.2934 + 0.2125$
$= 0.5059$

2 (a) $P(Z > p) = 0.123$
$p = 1.16$
$P(0.28 < Z < p) = P(0.28 < Z < 1.16)$
$= P(Z > 0.28) - P(Z > 1.16)$
$= 0.3897 - 0.123$
$= 0.2667$



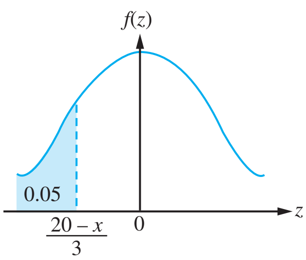
(b) $\mu = 20, \sigma = 3$

(i) $Z = 1.8$
$\frac{x - 20}{3} = 1.8$
$x = 25.4$ hours



(ii) $P(15 < X < 28)$
$= P\left(\frac{15 - 20}{3} < Z < \frac{28 - 20}{3}\right)$
$= P(-1.667 < Z < 2.667)$
$= 1 - P(Z < -1.667) - P(Z > 2.667)$
$= 1 - P(Z > 1.667) - P(Z > 2.667)$
$= 1 - 0.04778 - 0.00301$
$= 0.9492$

(iii) $P(X < x) < 0.05$
$P\left(Z < \frac{x - 20}{3}\right) < 0.05$
$P\left(Z > \frac{20 - x}{3}\right) = 0.05$
$\frac{20 - x}{3} = 1.645$
$x = 15.07$ hours



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3 (a) Number of packets rejected
= 10 000(0.03)
= 300

(b) $\mu = 550$, $\sigma = 22$

$$P(X < k) = 0.01$$
$$P\left(Z < \frac{k - 550}{22}\right) = 0.01$$
$$P\left(Z > \frac{550 - k}{22}\right) = 0.01$$
$$\frac{550 - k}{22} = 2.326$$
$$k = 498.83 \text{ g}$$
$$P(X > m) = 0.02$$
$$P\left(Z > \frac{m - 550}{22}\right) = 0.02$$
$$\frac{m - 550}{22} = 2.054$$
$$m = 595.188 \text{ g}$$

Thus, the range of mass for each packet to not get rejected
$= 498.83 < x < 595.19$

(c)
$$P(X > x) = 0.05$$
$$P\left(Z > \frac{x - 550}{22}\right) = 0.05$$
$$\frac{x - 550}{22} = 1.645$$
$$x = 586.2 \text{ g}$$

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