FORM 5

CHAPTER 1 Circular Measure

Self Test 1

1 $0.65 \text{ rad} = \frac{0.65}{3.142} \times 180°$
$= 37°14'$

2 $75° = \frac{75°}{180°} \times 3.142$
$= 1.31 \text{ rad}$

3 $120° = \frac{120°}{180°} \times \pi$
$= \frac{2}{3} \times \pi$
$\therefore k = \frac{2}{3}$

Self Test 2

1 (a) $DE = 1.5$
$\alpha(2) = 1.5$
$\alpha = 0.75 \text{ rad}$

(b) $\cos \alpha = \frac{AB}{AC}$
$\cos\left(0.75^r\right) = \frac{4}{AC}$
$\cos(42.97°) = \frac{4}{AC}$
$AC = \frac{4}{0.7318}$
$= 5.47 \text{ cm}$
$\therefore DC = 5.47 - 2 = 3.47 \text{ cm}$
$CB = \sqrt{5.47^2 - 4^2} = 3.73 \text{ cm}$

Perimeter of the shaded region $= \overset{\frown}{DE} + EB + CB + CD$
$= 1.5 + 2 + 3.73 + 3.47$
$= 10.7 \text{ cm}$

2 $\theta j = 33.6$
$\alpha(5.6) = 33.6$
$\alpha = 6 \text{ rad}$
Major angle $AOB = 6 \text{ rad}$
$\therefore$ Minor angle $AOB = 2\pi - 6$
$= 2(3.142) - 6$
$\theta = 0.284 \text{ rad}$

3 Radius $= OB = OA = 8 \text{ cm}$
(a) Arc length $AB = 0.92(8)$
$= 7.36 \text{ cm}$

(b) $PB = \sqrt{8^2 - 6^2}$
$= \sqrt{28}$
$= 5.29 \text{ cm}$
Perimeter of the shaded region $= 2 + 7.36 + 5.29$
$= 14.65 \text{ cm}$

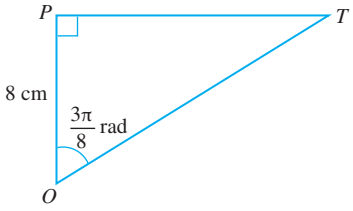
Self Test 3

1 Perimeter of segment $= \overset{\frown}{AB} + AB$
$AB = \sqrt{r^2 + r^2 - 2(r)(r)\cos\left(\frac{\pi}{3}\right)^r}$
$= \sqrt{2r^2 - 2r^2\cos 60°}$
$= \sqrt{2r^2 - 2r^2\left(\frac{1}{2}\right)}$
$= \sqrt{r^2}$
$= r \text{ cm}$
Arc length $AB = \frac{\pi}{3} r \text{ cm}$

Perimeter of segment $= \frac{\pi}{3} r + r$
$26 = r\left(\frac{\pi}{3} + 1\right)$
$r = 12.7 \text{ cm}$

Area of the shaded region $= \frac{1}{2}\left(\frac{\pi}{3}\right)12.7^2 - \frac{1}{2}(12.7)^2 \sin\left(\frac{\pi}{3}\right)^r$
$= \frac{1}{2}\left(\frac{\pi}{3}\right)12.7^2 - \frac{1}{2}(12.7)^2 \sin 60°$
$= \frac{1}{2}\left(\frac{\pi}{3}\right)12.7^2 - \frac{1}{2}(12.7)^2\left(\frac{\sqrt{3}}{2}\right)$
$= 84.46 - 69.84$
$= 14.62 \text{ cm}^2$

2 (a) $\angle POT = \frac{3\pi}{4} \div 2$
$= \frac{3\pi}{8} \text{ rad}$
$\tan\frac{3\pi}{8} = \frac{PT}{8}$
$PT = 2.414(8)$
$= 19.3 \text{ cm}$



(b) Area of the shaded region $= \frac{1}{2}(8)(19.3) \times 2 - \frac{1}{2}\left(\frac{3\pi}{4}\right)(8^2)$
$= 154.4 - 75.408$
$= 78.99 \text{ cm}^2$

3 Major angle $AOB = 2\pi - 2.2 = 4.084 \text{ rad}$
(a) Perimeter of the shaded region $= 4.084(6) + 6 + 6$
$= 36.5 \text{ cm}$

(b) Area of $\triangle AOB = \frac{1}{2}(6)(6)\sin 2.2^r$
$= \frac{1}{2}(6)(6)\sin 126.03°$
$= 14.557 \text{ cm}^2$

Area of the shaded region $= \frac{1}{2}(4.084)(6^2)$
$= 73.51 \text{ cm}^2$

Ratio $= 73.51 : 14.553$
$= \frac{73.51}{14.557} : 1$
$= 5.05 : 1$
$= 5 : 1$

Self Test 4

1 $\angle BOC = 2\pi - 1.7 - 2.4$
$= 2.18$ rad

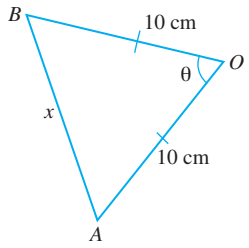
Arc length $AB = 1.7(12) = 20.4$ cm

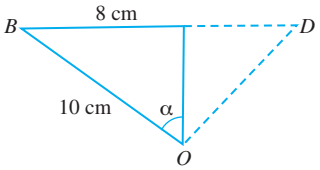
$BC = \sqrt{12^2 + 12^2 - 2(12)(12)\cos(2.18)^r}$
$= \sqrt{12^2 + 12^2 - 2(12)(12)\cos 124.9°}$
$= \sqrt{12^2 + 12^2 - 2(12)(12)(-0.5721)}$
$= \sqrt{452.76}$
$= 21.28$ cm

Perimeter of the shaded region $= OA + \overset{\frown}{AB} + BC + OC$
$= 12 + 20.4 + 21.28 + 12$
$= 65.68$ cm

Area of the shaded region $= \frac{1}{2}(1.7)(12^2) + \frac{1}{2}(12)(12)\sin(2.18)^r$
$= \frac{1}{2}(1.7)(12^2) + \frac{1}{2}(12)(12)\sin 124.9°$
$= 122.4 + 59.05$
$= 181.45$ cm^2

2 (a)





$\sin\alpha = \frac{8}{10}$
$\alpha = 0.927$ rad

$\theta = \frac{2\pi - 2(0.927)}{2} = 2.215$ rad

$AB = \sqrt{10^2 + 10^2 - 2(10)(10)\cos(2.215)^r}$
$= \sqrt{10^2 + 10^2 - 2(10)(10)\cos 126.9°}$
$= \sqrt{10^2 + 10^2 - 2(10)(10)(-0.6)}$
$= \sqrt{320}$
$= 17.9$ cm (Shown)

(b) $\angle BAO = \frac{\pi - 2.215}{2}$
$= 0.4635$ rad

$\therefore \angle BAD = 2(0.4635) = 0.927$ cm

Arc length $BED = 0.927(17.9)$
$= 16.59$ cm

Arc length $BCD = (2 \times 0.927)(10)$
$= 18.54$ cm

Perimeter of the shaded region $= 16.59 + 18.54$
$= 35.13$ cm

Area of sector $BODC = \frac{1}{2}(1.854)(10^2)$
$= 92.7$ cm^2

Area of $\triangle BOD = \frac{1}{2}(10)(10)\sin(1.854)^r$
$= \frac{1}{2}(10)(10)\sin 106.21°$
$= \frac{1}{2}(10)(10)(0.9602)$
$= 48$ cm^2

Area of segment BED
$=$ Area of sector $BAED$ – Area of $\triangle ABD$
$= \frac{1}{2}(0.927)(17.9^2) - \frac{1}{2}(17.9)(17.9)\sin(0.927)^r$
$= \frac{1}{2}(0.927)(17.9^2) - \frac{1}{2}(17.9)(17.9)\sin 53.11°$
$= 148.51 - 128.13$
$= 20.38$ cm^2

Area of the shaded region
$=$ Area of sector $BODC$ – Area of $\triangle BOD$ –
Area of segment BED
$= 92.7 - 48 - 20.38$
$= 24.32$ cm^2

3 (a) $\cos\angle ACB = \frac{5^2 + 5^2 - 8^2}{2(5)(5)}$
$= -0.28$
$\angle ACB = 1.855$ rad
$\therefore \angle BCE = \pi - 1.855$
$= 1.287$ rad (Shown)

(b) (i) $DE = 5 - (8 - 5) = 2$ cm

$\angle DAB = \frac{\pi - 1.855}{2}$
$= 0.6435$ cm

$\overset{\frown}{BD} = 0.6435(8)$
$= 5.148$ cm

$\overset{\frown}{BE} = 1.287(5)$
$= 6.435$ cm

Perimeter of the shaded region $= \overset{\frown}{BD} + \overset{\frown}{BE} + DE$
$= 5.148 + 6.435 + 2$
$= 13.583$ cm

(ii) Area of sector $BCE = \frac{1}{2}(1.287)(5^2)$
$= 16.0875$ cm^2

Area of sector $ABD = \frac{1}{2}(0.6435)(8^2)$
$= 20.592$ cm^2

Area of $\triangle ACB = \frac{1}{2}(5)(5)\sin(1.855)^r$
$= \frac{1}{2}(5)(5)\sin 106.27°$
$= 12$ cm^2

Area of segment $BCD = 20.592 - 12$
$= 8.597$ cm^2

$\therefore$ Area of the shaded region $= 16.0875 - 8.597$
$= 7.49$ cm^2

SPM Practice

Paper 1

1 (a) Arc length $BC = 16$
$1.2r = 16$
$r = 13.33$

(b) Area of the shaded region $=$ Area of sector BOC –
Area of sector ABC

$65 = \frac{1}{2}(\angle BOC)(18^2) - \frac{1}{2}(1.2)(13.33)^2$
$162(\angle BOC) = 65 + 106.613$
$\angle BOC = \frac{171.613}{162}$
$= 1.06$ rad

2 $OD : OA = 3 : 7$

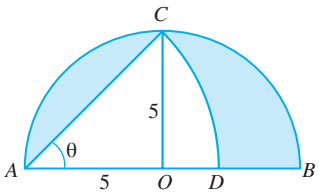
Area of I $= \frac{1}{2}\theta(3j)^2 - \frac{1}{2}(3j)^2\sin\theta$
$= \frac{9\theta j^2}{2} - \frac{9j^2\sin\theta}{2}$
$= 9\left(\frac{\theta j^2}{2} - \frac{j^2\sin\theta}{2}\right)$

Area of II $= \frac{1}{2}\theta(7j)^2 - \frac{1}{2}(7j)^2\sin\theta$
$= \frac{49\theta j^2}{2} - \frac{49j^2\sin\theta}{2}$
$= 49\left(\frac{\theta j^2}{2} - \frac{j^2\sin\theta}{2}\right)$

Area of I : Area of II $= 9\left(\frac{\theta j^2}{2} - \frac{j^2 \sin\theta}{2}\right) : 49\left(\frac{\theta j^2}{2} - \frac{j^2 \sin\theta}{2}\right)$

$= 9 : 49$ (Shown)

3



Radius $AC = \sqrt{5^2 + 5^2}$

$= 5\sqrt{2}$ cm

Angle $CAB = \theta$

$\tan\theta = \frac{5}{5}$

$\theta = \tan^{-1}(1)$

$= 45°$

$= \frac{\pi}{4}$ rad

$\therefore$ Area of shaded region $= \frac{1}{2}\left(\frac{2\pi}{2}\right)(5^2) - \frac{1}{2}\left(\frac{\pi}{4}\right)(5\sqrt{2})^2$

$= \frac{25\pi}{2} - \frac{25\pi}{4}$

$= \frac{25\pi}{4}$ cm^2

4 (a) Area of circle $ADE = \pi(5)^2 = 25\pi$ cm^2

Area of sector $BOC = \frac{1}{2}\theta(10)^2 = 50\theta$ cm^2

$50\theta = 25\pi$

$\theta = \frac{\pi}{2}$ rad

(b) Area of major sector $AOD = \frac{1}{2}\left(\frac{3\pi}{2}\right)(5^2)$

$= \frac{75}{4}\pi$ cm^2

Area of shaded region = Area of major sector AOD + (Area of sector BOC – Area of sector AOD)

$= \frac{75}{4}\pi + \left[\frac{1}{2} \times \frac{\pi}{2} \times (10^2 - 5^2)\right]$

$= \frac{75}{4}\pi + \frac{75}{4}\pi$

$= \frac{75}{2}\pi$ cm^2

5 $\overset{\frown}{PQ} : \overset{\frown}{QR} = 4 : 3$

(a) $\overset{\frown}{PQ} = \left(\frac{\pi}{2} - \theta\right)(15)$

$= \frac{15}{2}\pi - \frac{15}{2}\theta$

$\overset{\frown}{QR} = 15\theta$

$\frac{\overset{\frown}{PQ}}{\overset{\frown}{QR}} = \frac{4}{3}$

$3\left(\frac{15}{2}\pi - \frac{15}{2}\theta\right) = 4(15\theta)$

$\frac{45}{2}\pi - \frac{45}{2}\theta = 60\theta$

$\frac{45}{2}\pi = \frac{165}{2}\theta$

$\theta = \frac{3}{11}\pi$ rad

$= 0.857$ rad

(b) Area of shaded region

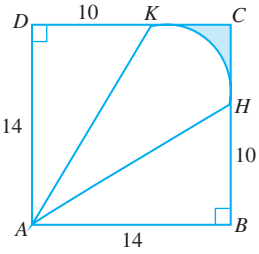
$= \frac{1}{2}\left(\frac{\pi}{2}\right)(15^2) - \frac{1}{2}(15)(15)\sin\left(\frac{\pi}{2} - 0.857\right)^r$

$= \frac{1}{2}\left(\frac{\pi}{2}\right)(15^2) - \frac{1}{2}(15)(15)\sin 40.9°$

$= 176.74 - 73.67$

$= 103.07$ cm^2

6



$DK : KC = 5 : 2$

$DK = \frac{5}{7}(14) = 10$ cm

$\therefore BH = 10$ cm, $CH = 4$ cm

(a) $\angle DAK = \angle HAB$

$= \tan^{-1}\left(\frac{10}{14}\right)$

$= 0.6204$ rad

$\therefore \angle HAK = \frac{\pi}{2} - 2(0.6204)$

$= 0.33$ rad

(b) $AH = \sqrt{14^2 + 10^2} = 17.2$ cm

$\therefore \overset{\frown}{KH} = 0.33(17.2) = 5.68$ cm

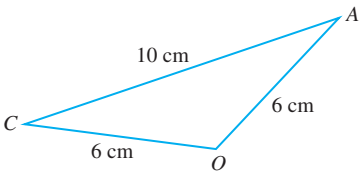
(c) Perimeter of shaded region $= \overset{\frown}{KH} + CH + KC$

$= 5.68 + 4 + 4$

$= 13.68$ cm

Paper 2

1 (a)



Let $\angle ACO = \theta$

$\cos\theta = \frac{10^2 + 6^2 - 6^2}{2(10)(6)}$

$= 0.833$

$\theta = 0.586$ rad

Angle $ACB = 2(0.586)$

$= 1.172$ rad

(b) Area of sector $OAXB = \frac{1}{2}(2.4)(6^2)$

$= 43.2$ cm^2

Area of $OAYB$ = Area of sector ACB – Area of $\triangle OAC$ – Area of $\triangle OCB$

$= \frac{1}{2}(1.172)(10^2) - 2 \times \frac{1}{2}(6)(10)\sin(0.586)^r$

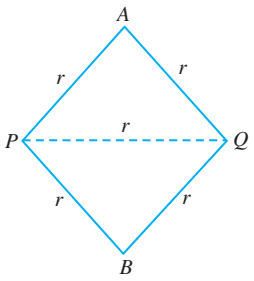
$= \frac{1}{2}(1.172)(10^2) - (6)(10)\sin 33.58°$

$= 58.6 - 33.18$

$= 25.42$ cm^2

$\therefore$ Area of shaded region $= 43.2 - 25.42 = 17.78$ cm^2

2



(a) $\triangle APQ$ and $\triangle BPQ$ are equilateral triangles

$\angle APQ = \angle QPB$

$= 60°$

$= \frac{\pi}{3}$rad

$\angle APB = 2\left(\frac{\pi}{3}\right)$

$= \frac{2\pi}{3}$ rad (Shown)

(b) Area of segment AQ = Area of sector APQ – Area of $\triangle APQ$

$= \frac{1}{2}\left(\frac{\pi}{3}\right)(r^2) - \frac{1}{2}(r)(r)\sin\left(\frac{\pi}{3}\right)^r$

$= \frac{1}{2}\left(\frac{\pi}{3}\right)(r^2) - \frac{1}{2}(r)(r)\sin 60°$

$= \frac{\pi r^2}{6} - \frac{\sqrt{3}}{4}r^2$

Area of shaded region $= 4\left(\frac{\pi r^2}{6} - \frac{\sqrt{3}}{4}r^2\right)$

$= 2r^2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)$ (Shown)

(c) Length of $AP = AQ = PB = QB = r$ cm

$\overset{\frown}{AQ} = \overset{\frown}{AP} = \overset{\frown}{PB} = \overset{\frown}{QB}$

$= \frac{\pi}{3}(r)$

$= \frac{\pi r}{3}$

Perimeter of shaded region $= 4r + 4\left(\frac{\pi r}{3}\right)$

$= 4r + \frac{4\pi r}{3}$

$= 4r\left(1 + \frac{\pi}{3}\right)$ (Shown)

3 (a) $\overset{\frown}{AB} = \frac{\pi}{3}(8) = \frac{8\pi}{3}$ cm

Perimeter of shaded region $= \frac{8\pi}{3} + \frac{8\pi}{3} + 4(8)$

$= \left(\frac{16\pi}{3} + 32\right)$ cm

(b) Area of sector $PAB = \frac{1}{2}\left(\frac{\pi}{3}\right)(8^2) = \frac{32\pi}{3}$ cm^2

Area of segment $AB = \frac{32}{3}\pi - \frac{1}{2}(8)(8)\sin\left(\frac{\pi}{3}\right)^r$

$= \frac{32}{3}\pi - \frac{1}{2}(8)(8)\sin 60°$

$= \frac{32}{3}\pi - 16\sqrt{3}$

$= 5.8$ cm^2

Area of shaded region $PAB = \frac{32}{3}\pi - 2(5.8)$

$= 21.91$ cm^2

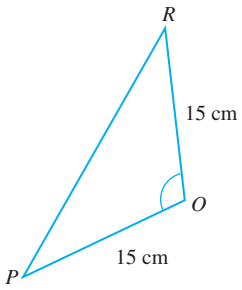
Area of the whole shaded region

= Area of shaded region PAB + Area of shaded region AQB

$= 2(21.91)$

$= 43.82$ cm^2

4



(a) $\angle PRQ = 0.45$ rad

$\angle POQ = 2(0.45) = 0.9$ rad

$\angle POR = \frac{2\pi - 0.9}{2} = 2.69$ rad

$PR^2 = 15^2 + 15^2 - 2(15)(15)\cos(2.69)^r$

$= 15^2 + 15^2 - 2(15)(15)\cos 154.11°$

$= 854.9$

$PR = 29.24$ cm $= RQ$

(b) Major angle $PRQ = 2\pi - 0.45$

$= 5.834$ rad

Major arc length $PRQ = 5.834(29.24)$

$= 170.59$ cm

(c) $\overset{\frown}{PSQ} = 0.45(29.24) = 13.158$ cm

$\overset{\frown}{PTQ} = 0.9(15) = 13.5$ cm

Perimeter of shaded region $= 13.5 + 13.158$

$= 26.66$ cm

(d) Area of sector $OPTQ = \frac{1}{2}(0.9)(15^2)$

$= 101.25$ cm^2

Area of $OPSQ$ = Area of sector PRQ – Area of $\triangle OPR$ – Area of $\triangle OQR$

$= \frac{1}{2}(0.45)(29.24)^2 - 2 \times \frac{1}{2}(15)(15)\sin(2.69)^r$

$= \frac{1}{2}(0.45)(29.24)^2 - 2 \times \frac{1}{2}(15)(15)\sin 154.11°$

$= 192.37 - 98.24$

$= 94.13$ cm^2

Area of shaded region $= 101.25 - 94.13$

$= 7.12$ cm^2