Penyelesaian Lengkap

PRAKTIS 8

Kertas 1

Bahagian A

1 (a) $\overrightarrow{PQ}$ selari dengan/*parallel to* $\overrightarrow{QR}$

$$\overrightarrow{PQ} = \lambda\overrightarrow{QR}$$
$$\lambda(12\underset{\sim}{a} - 6\underset{\sim}{b}) = 8\underset{\sim}{a} + (4k - 3)\underset{\sim}{b}$$
$$12\lambda\underset{\sim}{a} - 6\lambda\underset{\sim}{b} = 8\underset{\sim}{a} + (4k - 3)\underset{\sim}{b}$$

Bandingkan vektor $\underset{\sim}{a}$,
Compare vector $\underset{\sim}{a}$,
$$12\lambda = 8$$
$$\lambda = \frac{2}{3}$$

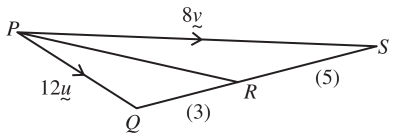
Bandingkan vektor $\underset{\sim}{b}$,
Compare vector $\underset{\sim}{b}$,
$$-6\lambda = 4k - 3$$

Gantikan/*Substitute* λ,
$$-6\left(\frac{2}{3}\right) = 4k - 3$$
$$-4 = 4k - 3$$
$$4k = -1$$
$$k = -\frac{1}{4}$$

(b) $\dfrac{PQ}{QR} = \dfrac{2}{3}$

$PQ : QR = 2 : 3$

2



(a)
$$\overrightarrow{PQ} + \overrightarrow{QS} = \overrightarrow{PS}$$
$$12\underset{\sim}{u} + \overrightarrow{QS} = 8\underset{\sim}{v}$$
$$\overrightarrow{QS} = 8\underset{\sim}{v} - 12\underset{\sim}{u}$$

(b)
$$\overrightarrow{RS} = \frac{5}{3+5}(8\underset{\sim}{v} - 12\underset{\sim}{u})$$
$$= \frac{5}{8} \times 4(2\underset{\sim}{v} - 3\underset{\sim}{u})$$
$$= \frac{5}{2}(2\underset{\sim}{v} - 3\underset{\sim}{u})$$

$$\overrightarrow{PR} = \overrightarrow{PS} - \overrightarrow{RS}$$
$$= 8\underset{\sim}{v} - \frac{5}{2}(2\underset{\sim}{v} - 3\underset{\sim}{u})$$
$$= \frac{1}{2}(16\underset{\sim}{v} - 10\underset{\sim}{v} + 15\underset{\sim}{u})$$
$$= \frac{1}{2}(6\underset{\sim}{v} + 15\underset{\sim}{u})$$
$$= 3\underset{\sim}{v} + \frac{15}{2}\underset{\sim}{u}$$

3 (a)
$$\underset{\sim}{p} + \underset{\sim}{q} = \begin{pmatrix} h-3 \\ -7 \end{pmatrix} + \begin{pmatrix} h \\ 7 \end{pmatrix}$$
$$= \begin{pmatrix} 2h-3 \\ 0 \end{pmatrix}$$

(b)
$$|\underset{\sim}{p} + \underset{\sim}{q}| = 7$$
$$\sqrt{(2h-3)^2 + (0)^2} = 7$$
$$(2h-3)^2 + 0 = 7^2$$
$$4h^2 - 12h + 9 = 49$$
$$4h^2 - 12h - 40 = 0$$
$$h^2 - 3h - 10 = 0$$
$$(h+2)(h-5) = 0$$
$$h + 2 = 0,\ h - 5 = 0$$
$$h = -2,\quad h = 5$$

4 (a)
$$\overrightarrow{DC} = \overrightarrow{DO} + \overrightarrow{OC}$$
$$= (-3\underset{\sim}{i} + 4\underset{\sim}{j}) + (-5\underset{\sim}{i} + 2\underset{\sim}{j})$$
$$= -8\underset{\sim}{i} + 6\underset{\sim}{j}$$

(b) Vektor unit dalam arah $\overrightarrow{DC}$
Unit vector in the direction of $\overrightarrow{DC}$
$$= \frac{1}{\sqrt{(-8)^2 + (6)^2}}(-8\underset{\sim}{i} + 6\underset{\sim}{j})$$
$$= \frac{1}{\sqrt{100}} \times 2(-4\underset{\sim}{i} + 3\underset{\sim}{j})$$
$$= \frac{1}{10} \times 2(-4\underset{\sim}{i} + 3\underset{\sim}{j})$$
$$= \frac{1}{5}(-4\underset{\sim}{i} + 3\underset{\sim}{j})$$

5 (a)
$$3h - 2 = 0$$
$$h = \frac{2}{3}$$
$$6h + 2k = 0$$
$$6\left(\frac{2}{3}\right) + 2k = 0$$
$$2k = -4$$
$$k = -2$$

(b) (i)
$$\overrightarrow{OE} = \overrightarrow{GF} = 3\underset{\sim}{i} + 5\underset{\sim}{j}$$
$$\overrightarrow{OG} + \overrightarrow{GE} = \overrightarrow{OE}$$
$$(8\underset{\sim}{i} + 3\underset{\sim}{j}) + \overrightarrow{GE} = (3\underset{\sim}{i} + 5\underset{\sim}{j})$$
$$\overrightarrow{GE} = (3\underset{\sim}{i} + 5\underset{\sim}{j}) - (8\underset{\sim}{i} + 3\underset{\sim}{j})$$
$$= 3\underset{\sim}{i} + 5\underset{\sim}{j} - 8\underset{\sim}{i} - 3\underset{\sim}{j}$$
$$= -5\underset{\sim}{i} + 2\underset{\sim}{j}$$

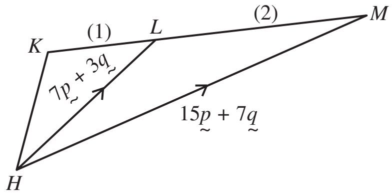
(ii)
$$|\overrightarrow{GE}| = \sqrt{(-5)^2 + (2)^2}$$
$$= \sqrt{29} \text{ unit/}\textit{units}$$

6 (a)
$$2\underset{\sim}{u} - \underset{\sim}{v} = 2(8\underset{\sim}{i} + 2\underset{\sim}{j}) - (12\underset{\sim}{i} + k\underset{\sim}{j})$$
$$= 16\underset{\sim}{i} + 4\underset{\sim}{j} - 12\underset{\sim}{i} - k\underset{\sim}{j}$$
$$= 4\underset{\sim}{i} + 4\underset{\sim}{j} - k\underset{\sim}{j}$$
$$= 4\underset{\sim}{i} + (4 - k)\underset{\sim}{j}$$

(b)
$$|2\underset{\sim}{u} - \underset{\sim}{v}| = \sqrt{17}$$
$$|4\underset{\sim}{i} + (4-k)\underset{\sim}{j}| = \sqrt{17}$$
$$\sqrt{(4)^2 + (4-k)^2} = \sqrt{17}$$
$$16 + (4-k)^2 = 17$$
$$16 + 16 - 8k + k^2 = 17$$
$$k^2 - 8k + 15 = 0$$
$$(k-3)(k-5) = 0$$
$$k - 3 = 0,\ k - 5 = 0$$
$$k = 3,\quad k = 5$$

Bahagian B

7 (a) (i)



$KL : KM = 1 : 3 \Rightarrow KL : LM = 1 : 2$
$$\overrightarrow{HL} + \overrightarrow{LM} = \overrightarrow{HM}$$
$$(7\underset{\sim}{p} + 3\underset{\sim}{q}) + \overrightarrow{LM} = 15\underset{\sim}{p} + 7\underset{\sim}{q}$$
$$\overrightarrow{LM} = 8\underset{\sim}{p} + 4\underset{\sim}{q}$$
$$\overrightarrow{KL} = \frac{1}{2}\overrightarrow{LM}$$
$$= \frac{1}{2}(8\underset{\sim}{p} + 4\underset{\sim}{q})$$
$$= 4\underset{\sim}{p} + 2\underset{\sim}{q}$$

(ii)
$$\overrightarrow{HK} + \overrightarrow{KL} = \overrightarrow{HL}$$
$$\overrightarrow{HK} + (4\underset{\sim}{p} + 2\underset{\sim}{q}) = 7\underset{\sim}{p} + 3\underset{\sim}{q}$$
$$\overrightarrow{HK} = 3\underset{\sim}{p} + \underset{\sim}{q}$$

(b)
$$\overrightarrow{HK} = 3\underset{\sim}{p} + \underset{\sim}{q}$$
$$= 3(2\underset{\sim}{i} + \underset{\sim}{j}) + (2\underset{\sim}{i} + 3\underset{\sim}{j})$$
$$= 6\underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{i} + 3\underset{\sim}{j}$$
$$= 8\underset{\sim}{i} + 6\underset{\sim}{j}$$

(c)
$$\overrightarrow{HK} = 8\underset{\sim}{i} + 6\underset{\sim}{j}$$
$$\overrightarrow{HK} = \sqrt{8^2 + 6^2}$$
$$= \sqrt{100}$$
= 10 unit/*units*

8 (a) (i)
$$\overrightarrow{HG} = \overrightarrow{OF} = 10\underset{\sim}{i} + 4\underset{\sim}{j}$$
$$\overrightarrow{OH} + \overrightarrow{HG} = \overrightarrow{OG}$$
$$\overrightarrow{OH} + 10\underset{\sim}{i} + 4\underset{\sim}{j} = 14\underset{\sim}{i} + 10\underset{\sim}{j}$$
$$\overrightarrow{OH} = 4\underset{\sim}{i} + 6\underset{\sim}{j}$$

(ii) Vektor unit dalam arah $\overrightarrow{OH}$
Unit vector in the direction $\overrightarrow{OH}$
$$= \frac{1}{|\overrightarrow{OH}|}\overrightarrow{OH}$$
$$= \frac{1}{\sqrt{4^2 + 6^2}}(4\underset{\sim}{i} + 6\underset{\sim}{j})$$
$$= \frac{1}{\sqrt{52}}(4\underset{\sim}{i} + 6\underset{\sim}{j})$$
$$= \frac{1}{2 \times \sqrt{13}} \times 2(2\underset{\sim}{i} + 3\underset{\sim}{j})$$
$$= \frac{1}{\sqrt{13}}(2\underset{\sim}{i} + 3\underset{\sim}{j})$$

(b) (i)
$$\overrightarrow{FG} = \overrightarrow{OH} = 4\underset{\sim}{i} + 6\underset{\sim}{j}$$
$$\overrightarrow{FW} = \overrightarrow{FG} + \overrightarrow{GW}$$
$$= (4\underset{\sim}{i} + 6\underset{\sim}{j}) + (\underset{\sim}{i} - 4\underset{\sim}{j})$$
$$= 5\underset{\sim}{i} + 2\underset{\sim}{j}$$

(ii)
$$\overrightarrow{OW} = \overrightarrow{OG} + \overrightarrow{GW}$$
$$= (14\underset{\sim}{i} + 10\underset{\sim}{j}) + (\underset{\sim}{i} - 4\underset{\sim}{j})$$
$$= 15\underset{\sim}{i} + 6\underset{\sim}{j}$$
$$= 3(5\underset{\sim}{i} + 2\underset{\sim}{j})$$
$$= 3\overrightarrow{FW}$$

Maka, O, F dan W adalah segaris.
Thus, O, F and W are collinear.

Kertas 2

Bahagian A

1 (a)
$$\overrightarrow{OG} + \overrightarrow{GH} = \overrightarrow{OH}$$
$$\overrightarrow{OG} + \begin{pmatrix}3\\6\end{pmatrix} = \begin{pmatrix}5\\2\end{pmatrix}$$
$$\overrightarrow{OG} = \begin{pmatrix}5\\2\end{pmatrix} - \begin{pmatrix}3\\6\end{pmatrix}$$
$$= \begin{pmatrix}2\\-4\end{pmatrix}$$
$\therefore G(2, -4)$

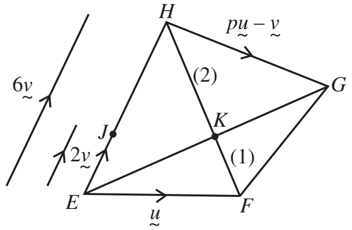
(b) Vektor unit dalam arah $\overrightarrow{OG}$
Unit vector in the direction of $\overrightarrow{OG}$
$$= \frac{1}{\sqrt{2^2 + (-4)^2}}\begin{pmatrix}2\\-4\end{pmatrix}$$
$$= \frac{1}{\sqrt{20}}\begin{pmatrix}2\\-4\end{pmatrix}$$
$$= \frac{1}{2\sqrt{5}} \times \begin{pmatrix}2\\-4\end{pmatrix}$$
$$= \frac{1}{\sqrt{5}}\begin{pmatrix}1\\-2\end{pmatrix}$$

(c)
$$\overrightarrow{PQ} = \lambda\overrightarrow{GH}$$
$$\begin{pmatrix}w\\8\end{pmatrix} = \lambda\begin{pmatrix}3\\6\end{pmatrix}$$
$w = 3\lambda$ … ①
$8 = 6\lambda$ … ②

Daripada/*From* ②,
$8 = 6\lambda$
$$\lambda = \frac{4}{3}$$

Daripada/*From* ①,
$$w = 3\left(\frac{4}{3}\right)$$
$$= 4$$

2



(a) (i) $\overrightarrow{EH} + \overrightarrow{HF} = \overrightarrow{EF}$

$3\overrightarrow{EJ} + \overrightarrow{HF} = \overrightarrow{EF}$

$6\underset{\sim}{v} + \overrightarrow{HF} = \underset{\sim}{u}$

$\overrightarrow{HF} = \underset{\sim}{u} - 6\underset{\sim}{v}$

(ii) $\overrightarrow{EK} = \overrightarrow{EH} + \overrightarrow{HK}$

$= \overrightarrow{EH} + \frac{2}{3}\overrightarrow{HF}$

$= 6\underset{\sim}{v} + \frac{2}{3}(\underset{\sim}{u} - 6\underset{\sim}{v})$

$= 6\underset{\sim}{v} + \frac{2}{3}\underset{\sim}{u} - 4\underset{\sim}{v}$

$= \frac{2}{3}\underset{\sim}{u} + 2\underset{\sim}{v}$

(b) $\overrightarrow{EH} = -\overrightarrow{HG} + \overrightarrow{EG}$

$6\underset{\sim}{v} + (p\underset{\sim}{u} - \underset{\sim}{v}) = \frac{1}{q}\overrightarrow{EK}$

$6\underset{\sim}{v} + (p\underset{\sim}{u} - \underset{\sim}{v}) = \frac{1}{q}\left(\frac{2}{3}\underset{\sim}{u} + 2\underset{\sim}{v}\right)$

$(p\underset{\sim}{u} - \underset{\sim}{v}) = \frac{1}{q}\left(\frac{2}{3}\underset{\sim}{u} + 2\underset{\sim}{v}\right) - 6\underset{\sim}{v}$

$p\underset{\sim}{u} - \underset{\sim}{v} = \frac{2}{3q}\underset{\sim}{u} + \frac{2}{q}\underset{\sim}{v} - 6\underset{\sim}{v}$

$p\underset{\sim}{u} - \underset{\sim}{v} = \frac{2}{3q}\underset{\sim}{u} + \left(\frac{2}{q} - 6\right)\underset{\sim}{v}$

$p = \frac{2}{3q}$ … ①

$\frac{2}{q} - 6 = -1$ … ②

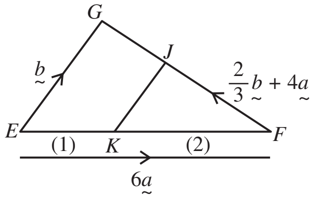
Daripada/*From* ②, $\frac{2}{q} = 5$

$q = \frac{2}{5}$

Daripada/*From* ①, $p = \frac{2}{3\left(\frac{2}{5}\right)}$

$p = \frac{5}{3}$

3



(a) $\overrightarrow{KF} = \frac{2}{3}\overrightarrow{EF}$

$= \frac{2}{3}(6\underset{\sim}{a})$

$= 4\underset{\sim}{a}$

Dalam/*In* ΔKJF,

$\overrightarrow{KJ} = \overrightarrow{KF} + \overrightarrow{FJ}$

$= 4\underset{\sim}{a} + \frac{2}{3}\underset{\sim}{b} - 4\underset{\sim}{a}$

$= \frac{2}{3}\underset{\sim}{b}$

$= \frac{2}{3}\overrightarrow{EG}$

$\therefore$ *EG* adalah selari dengan *KJ*.
EG is parallel to KJ.

(b) $\overrightarrow{KJ} = \frac{2}{3}\overrightarrow{EG}$

$|\overrightarrow{KJ}| = \sqrt{\left(\frac{2}{3}\right)^2} \times |\overrightarrow{EG}|$

$= \frac{2}{3}|\overrightarrow{EG}|$

$KJ : EG = 2 : 3$

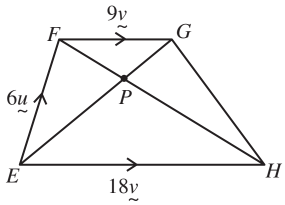
(c) Dalam/*In* ΔEJK,

$\overrightarrow{EJ} = \overrightarrow{EK} + \overrightarrow{KJ}$

$\overrightarrow{EJ} = \frac{1}{3}(6\underset{\sim}{a}) + \frac{2}{3}\underset{\sim}{b}$

$= 2\underset{\sim}{a} + \frac{2}{3}\underset{\sim}{b}$

4



(a) (i) $\overrightarrow{FH} = \overrightarrow{EH} - \overrightarrow{EF}$

$= 18\underset{\sim}{v} - 6\underset{\sim}{u}$

(ii) $\overrightarrow{EG} = \overrightarrow{EF} + \overrightarrow{FG}$

$= \overrightarrow{EF} + \frac{1}{2}\overrightarrow{EH}$

$= 6\underset{\sim}{u} + 9\underset{\sim}{v}$

(b) $\overrightarrow{FP} = \alpha\overrightarrow{FH}$ dan/*and* $\overrightarrow{PG} = \lambda\overrightarrow{EG}$, λ pemalar/*constant*

Dalam/*In* ΔFPG,

$\overrightarrow{FP} + \overrightarrow{PG} = \overrightarrow{FG}$

$\alpha\overrightarrow{FH} + \lambda\overrightarrow{EG} = \overrightarrow{FG}$

$\alpha(18\underset{\sim}{v} - 6\underset{\sim}{u}) + \lambda(6\underset{\sim}{u} + 9\underset{\sim}{v}) = 9\underset{\sim}{v}$

$18\alpha\underset{\sim}{v} - 6\alpha\underset{\sim}{u} + 6\lambda\underset{\sim}{u} + 9\lambda\underset{\sim}{v} = 9\underset{\sim}{v}$

$(-6\alpha + 6\lambda)\underset{\sim}{u} + (18\alpha + 9\lambda)\underset{\sim}{v} = 0\underset{\sim}{u} + 9\underset{\sim}{v}$

Bandingkan vektor/*Compare vectors,*

$-6\alpha + 6\lambda = 0$

$\lambda = \alpha$ … ①

$18\alpha + 9\lambda = 9$

$2\alpha + \lambda = 1$ … ②

Gantikan ① ke dalam ②,
Substitute ① *into* ②,

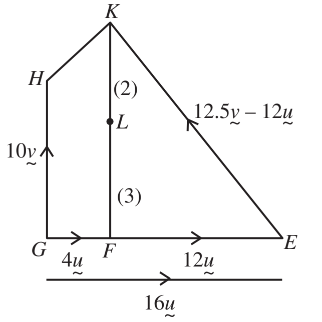
$2\alpha + \alpha = 1$

$3\alpha = 1$

$\alpha = \dfrac{1}{3}$

Bahagian B

5



(a) (i) $\overrightarrow{FE} = 3 \times 4\underset{\sim}{u}$

$= 12\underset{\sim}{u}$

$\overrightarrow{FK} = \overrightarrow{FE} + \overrightarrow{EK}$

$= 12\underset{\sim}{u} + (12.5\underset{\sim}{v} - 12\underset{\sim}{u})$

$= 12.5\underset{\sim}{v}$

(ii) $\overrightarrow{EH} = \overrightarrow{GH} - \overrightarrow{GE}$

$= 10\underset{\sim}{v} - 16\underset{\sim}{u}$

(b) $\overrightarrow{EL} = \overrightarrow{FL} - \overrightarrow{FE}$

$= \dfrac{3}{5}(12.5\underset{\sim}{v}) - 12\underset{\sim}{u}$

$= 7.5\underset{\sim}{v} - 12\underset{\sim}{u}$

$\overrightarrow{EH} = 10\underset{\sim}{v} - 16\underset{\sim}{u}$

$= \dfrac{4}{3}(7.5\underset{\sim}{v} - 12\underset{\sim}{u})$

$= \dfrac{4}{3}(\overrightarrow{EL})$

Maka, E, L dan H adalah segaris.
Thus, E, L and H are collinear.

(c) $\overrightarrow{EK} = 12.5\underset{\sim}{i} - 12\underset{\sim}{j}$

$|\overrightarrow{EK}| = \sqrt{(12.5)^2 + (-12)^2}$

$= \sqrt{300.25}$

Vektor unit dalam arah $\overrightarrow{EK}$
Unit vector in the direction $\overrightarrow{EK}$

$= \dfrac{1}{\sqrt{300.25}}(12.5\underset{\sim}{j} - 12\underset{\sim}{i})$

6 (a) $\overrightarrow{EB} = \dfrac{2}{3}\overrightarrow{ED} = \dfrac{2}{3}(24\underset{\sim}{v}) = 16\underset{\sim}{v}$

$\overrightarrow{EB} + \overrightarrow{BF} = \overrightarrow{EF}$

$16\underset{\sim}{v} + \overrightarrow{BF} = 24\underset{\sim}{u}$

$\overrightarrow{BF} = 24\underset{\sim}{u} - 16\underset{\sim}{v}$

$\overrightarrow{DG} = \dfrac{1}{2}\overrightarrow{BF}$

$= \dfrac{1}{2}(24\underset{\sim}{u} - 16\underset{\sim}{v})$

$= 12\underset{\sim}{u} - 8\underset{\sim}{v}$

$\overrightarrow{EG} = \overrightarrow{ED} + \overrightarrow{DG}$

$= 24\underset{\sim}{v} + (12\underset{\sim}{u} - 8\underset{\sim}{v})$

$= 12\underset{\sim}{u} + 16\underset{\sim}{v}$

(i) $\overrightarrow{EA} = \alpha\overrightarrow{EG}$

$= \alpha(12\underset{\sim}{u} + 16\underset{\sim}{v})$

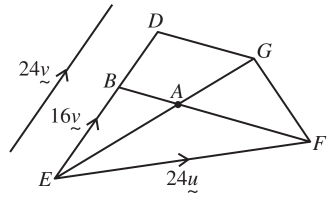
$= 12\alpha\underset{\sim}{u} + 16\alpha\underset{\sim}{v}$

(ii) $\overrightarrow{BA} = \beta\overrightarrow{BF}$

$= \beta(24\underset{\sim}{u} - 16\underset{\sim}{v})$

$= 24\beta\underset{\sim}{u} - 16\beta\underset{\sim}{v}$

(b)



Dalam/*In* ΔEBA,

$\overrightarrow{EB} + \overrightarrow{BA} = \overrightarrow{EA}$

$\overrightarrow{EB} + \beta\overrightarrow{BF} = \alpha\overrightarrow{EG}$

$16\underset{\sim}{v} + (24\beta\underset{\sim}{u} - 16\beta\underset{\sim}{v}) = 12\alpha\underset{\sim}{u} + 16\alpha\underset{\sim}{v}$

Bandingkan vektor $\underset{\sim}{u}$,
Compare vector $\underset{\sim}{u}$,

$24\beta = 12\alpha$ … ①

$2\beta = \alpha$

Bandingkan vektor $\underset{\sim}{v}$,
Compare vector $\underset{\sim}{v}$,

$16 - 16\beta = 16\alpha$

$1 - \beta = \alpha$ … ②

Gantikan ① ke dalam ②,
Substitute ① *into* ②,

$1 - \beta = 2\beta$

$1 = 3\beta$

$\beta = \dfrac{1}{3}$

Daripada/*From* ①,

$\alpha = 2\left(\dfrac{1}{3}\right)$

$= \dfrac{2}{3}$

(c) $\overrightarrow{DG} = 12\underset{\sim}{u} - 8\underset{\sim}{v}$

$\overrightarrow{BF} = 24\underset{\sim}{u} - 16\underset{\sim}{v}$

$= 2(12\underset{\sim}{u} - 8\underset{\sim}{v})$

$= 2\overrightarrow{DG}$

$|\overrightarrow{BF}| = 2|\overrightarrow{DG}|$

$\dfrac{|\overrightarrow{DG}|}{|\overrightarrow{BF}|} = \dfrac{1}{2}$

$\therefore DG : BF = 1 : 2$