Penyelesaian Lengkap

Praktis 5

Praktis Formatif

1 (a) Bilangan kanak-kanak berumur 5 hingga umur bawah 12 tahun yang diberi dua dos vaksin secara percuma.
The number of children aged from 5 *to below* 12 *years old who are given two doses of vaccine for free.*

(b) Kadar nadi setiap murid.
The heart rate of each student.

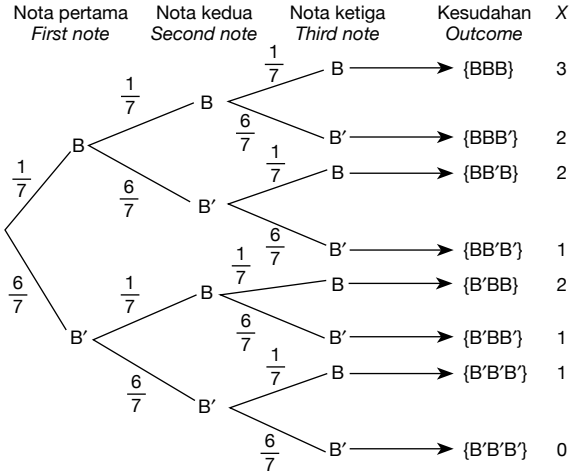
(c) Jumlah masa yang digunakan oleh pelajar untuk melayari Internet setiap minggu
The total time used weekly by the students in surfing the Internet.

2 (a) Pemboleh ubah rawak selanjar
Continuous random variable
$X = \{40 \text{ min}/mins \leqslant x \leqslant 60 \text{ min}/mins\}$

(b) Pemboleh ubah rawak diskret, $X = \{3, 4, 5\}$
Discrete random variable, $X = \{3, 4, 5\}$

3 (a) $X = \{0, 1, 2, 3\}$

(b)



B – Betul/*Correct*
B′ – Salah/*Wrong*

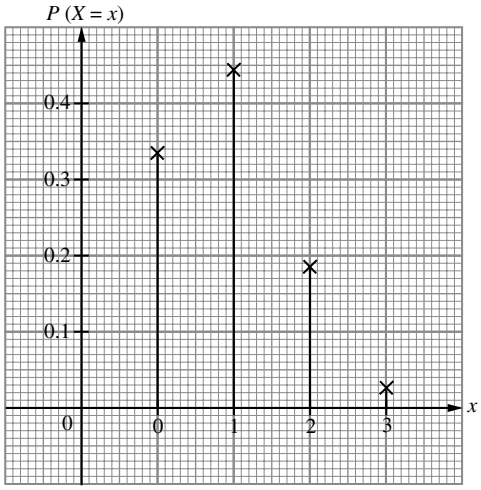
(c)

x	0	1	2	3
$P(X = x)$	$\frac{216}{343}$	$\frac{108}{343}$	$\frac{18}{343}$	$\frac{1}{343}$

4 (a)

x	0	1	2	3
$P(X = x)$	0.343	0.441	0.189	0.027

(b)



5 $X = \{0, 1, 2\}$
Pemboleh ubah rawak diskret kerana mempunyai nilai tersendiri yang boleh dibilang.
Discrete random variable because its value can be counted.

6 (a) Pemboleh ubah rawak diskret
Discrete random variable

(b) $X = \{0, 1, 2, 3\}$

(c) $2m = 1 - (0.462 + 0.322)$
$m = 0.108$

7 (a) Taburan binomial kerana kebarangkalian memperoleh bola dengan warna tertentu adalah tetap dan bilangan kali berjaya memperoleh bola dengan warna tertentu merupakan nilai tersendiri yang boleh dibilang.
Binomial distribution because the probability to get balls of certain colour is fix and the number of times of drawing balls of certain colour successfully is a discrete value that can be counted.

(b) Bukan taburan binomial kerana kebarangkalian memperoleh bola dengan warna tertentu berbeza kerana bergantung kepada kesudahan sebelumnya.
Not a binomial distribution as the probability to obtain balls of certain colour is different because it is dependent to the previous outcomes.

(c) Bukan taburan binomial kerana mempunyai lebih daripada dua kesudahan.
Not a binomial distribution because it has more than 2 *outcomes.*

8 $n = 10$

p = kebarangkalian pelajar menghadiri semua pengajaran dalam talian

= *the probability of students attend all the online lessons*

$= 0.85$

$q = 0.15$

(a) $P(X = 10) = {}^{10}C_{10}(0.85)^{10}\,(0.15)^{0}$

$= 0.1969$

(b) $P(X \geqslant 8)$

$= P(X = 8) + P(X = 9) + P(X = 10)$

$= {}^{10}C_{8}(0.85)^{8}\,(0.15)^{2} + {}^{10}C_{9}(0.85)^{9}\,(0.15)^{1} + {}^{10}C_{10}(0.85)^{10}\,(0.15)^{0}$

$= 0.2759 + 0.3474 + 0.1969$

$= 0.8202$

9 $n = 10$

p = kebarangkalian murid gagal mencapai minimum 50 markah

the probability of students who failed to achieve minimum 50 marks

$= 0.45$

$q = 0.55$

(a) $P(X = 2) = {}^{10}C_{2}\,(0.45)^{2}\,(0.55)^{8}$

$= 0.0763$

(b) Sekurang-kurangnya 8 orang murid mencapai minimum 50 markah = Selebih-lebihnya 2 orang murid gagal mencapai minimum 50 markah

At least 8 students score minimum 50 marks

= At most 2 students failed to score minimum 50 marks

$P(X \leqslant 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$= {}^{10}C_{0}\,(0.45)^{0}\,(0.55)^{10} + {}^{10}C_{1}\,(0.45)^{1}\,(0.55)^{9} + 0.0763$

$= 0.0025 + 0.0207 + 0.0763$

$= 0.0995$

10 $n = 10$

(a) $P(X = 0) = 0.9044$

${}^{10}C_{0}\,(p)^{0}\,(1-p)^{10} = 0.9044$

$\lg\,(1-p)^{10} = \lg 0.9044$

$10 \lg\,(1-p) = -0.04364$

$\lg\,(1-p) = -0.004364$

$1 - p = 10^{-0.004364}$

$p = 1 - 0.99$

$= 0.01$

(b) $P(X = 2) = {}^{10}C_{2}(0.01)^{2}(0.99)^{8}$

$= 0.0042$

11 (a) Kebarangkalian damak terkena papan damak pada tempat lebih dekat dengan *bullseye* daripada lilitan papan

Probability a dart hit the dart board at a place nearer to the bullseye than the circumference of the board

$$= \frac{\pi\left(\frac{1}{2}r\right)^{2}}{\pi r^{2}}$$

$$= \frac{1}{4}$$

$= 0.25$

Biar B peristiwa balingan damak yang lebih dekat dengan *bullseye* daripada lilitan papan

Let B the event of getting dart thrown that is nearer to the bullseye than the circumference of the board

(i) $P(B'B'B) = (0.75)(0.75)(0.25)$

$= 0.1406$

(ii) $P(B) + P(B'B) + P(B'B'B)$

$= (0.25) + (0.75)\,(0.25) + (0.75)(0.75)(0.25)$

$= 0.5781$

$n = ?,\ p = 0.25,\ P(X \geqslant 1) \geqslant 0.85$

$1 - P\,(X = 0) \geqslant 0.85$

$1 - 0.85 \geqslant P(X = 0)$

$0.15 \geqslant {}^{n}C_{0}(0.25)^{0}\,(0.75)^{n}$

$(0.75)^{n} \leqslant 0.15$

$n \lg 0.75 \leqslant \lg 0.15$

$$n \geqslant \frac{\lg 0.15}{\lg 0.75}$$

$n \geqslant 6.59$

$n_{\min} = 7$

(b) p = kebarangkalian mendapat telur rosak

probability of rotten egg obtained

$$= \frac{2}{30}$$

$$= \frac{1}{15}$$

$P(X \geqslant 1) > 0.9$

$1 - P(X = 0) > 0.9$

$1 - 0.9 > {}^{n}C_{0}\left(\frac{1}{15}\right)^{0}\left(\frac{14}{15}\right)^{n}$

$0.1 > \left(\frac{14}{15}\right)^{n}$

$\log 0.1 > \log\left(\frac{14}{15}\right)^{n}$

$\log 0.1 > n \log\left(\frac{14}{15}\right)$

$$\frac{\log 0.1}{\log\left(\frac{14}{15}\right)} < n$$

$n > 33.37$

$n_{\min} = 34$

(i) $n = 15,\ P(X = 0) = {}^{15}C_{0}\left(\frac{1}{15}\right)^{0}\left(\frac{14}{15}\right)^{15}$

$= 0.3553$

(ii) $P(X = 1) = {}^{15}C_{1}\left(\frac{1}{15}\right)^{1}\left(\frac{14}{15}\right)^{14}$

$= 0.3806$

12 (a) $n = 4$

(b) $P(X \leqslant 2) = 0.316 + 0.422 + 0.211$

$= 0.949$

$= 94.9\%$

(c) $P(X = 0) = 0.316$

${}^{4}C_{0}(p)^{0}\,(1-p)^{4} = 0.316$

$4 \log\,(1-p) = \log 0.316$ atau/*or* $1 - p = 0.316^{\frac{1}{4}}$

$\log\,(1-p) = -0.125$ $\quad 1 - 0.316^{\frac{1}{4}} = p$

$1 - p = 10^{-0.125}$ $\quad p = 0.25$

$p = 1 - 0.7499$

$p = 0.25$

(d) Bilangan penduduk/*number of residents*

$= 1.55 \text{ mil} \times 0.25$

$= 387\,500$

13 $X \sim B(3, p)$

(a) $P(X = 0) + P(X > 2) = 1 - r - s$

(b) $P(X = 0) = \dfrac{64}{125}$

$${}^3C_0\, p^0 (1-p)^3 = \frac{64}{125}$$

$$(1-p)^3 = \left(\frac{4}{5}\right)^3$$

$$1 - p = \frac{4}{5}$$

$$p = \frac{1}{5}$$

14 $p = \dfrac{1}{2}$, $P(X \geqslant 1) \geqslant 0.99$

$$1 - P(X = 0) \geqslant 0.99$$

$$P(X = 0) \leqslant 1 - 0.99$$

$${}^nC_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^n \leqslant 0.01$$

$$n \log\left(\frac{1}{2}\right) \leqslant \log 0.01$$

$$n \geqslant 6.6439$$

$$n_{\min} = 7$$

15 $X \sim \text{B}(50, 0.03)$

$P(X > 1) = 1 - [P(X = 0) + P(X = 1)]$

$= 1 - [{}^{50}C_0(0.03)^0(0.97)^{50} + {}^{50}C_1(0.03)^1(0.97)^{49}]$

$= 0.4447$

$= 0.445$ (3 sf)

Min/*Mean* $= np$

$= 50\ (0.03)$

$= 1.5$

Sisihan piawai/*Standard deviation* $= \sqrt{npq}$

$= \sqrt{50(0.03)(0.97)}$

$= 1.206$

16 p = kebarangkalian bahawa Eva dapat menghadirkan diri ke latihan

probability that Eva is able to attend the training

$= 0.8$

$n = 4$

(a) $P(X \geqslant 3) = P(X = 3) + P(X = 4)$

$= {}^4C_3(0.8)^3\ (0.2) + {}^4C_4(0.8)^4\ (0.2)^0$

$= 0.4096 + 0.4096$

$= 0.8192$

$X \sim (6, 0.8192)$

(b) $P(X \geqslant 5) = P(X = 5) + P(X) = 6)$

$= {}^6C_5(0.8192)^5\ (0.1808)^1 +$

$\quad {}^6C_6(0.8192)^6\ (0.1808)^0$

$= 0.4002 + 0.3022$

$= 0.7024$

17 (a) (i) Min/*Mean*, $\mu = 15$

(ii) $P(10 < X < 20)$

(iii) $P(X > 20) = \dfrac{1 - 0.68}{2}$

$= 0.16$

(b) $60 - 35 = 85 - 60 \qquad \therefore P(X > 85) = 0.16$

$P(35 < X < 85) = 1 - 2(0.16)$

$= 0.68$

$= 68\%$

68% daripada data terletak antara min $\pm\ 1\sigma$

68% *of the data lies between mean* $\pm\ 1\sigma$

$\therefore \quad 85 = 60 + \sigma$

$\sigma = 25$

(c) (i) 25 min/*mins* $< x <$ 35 min/*mins*

(ii) 20 min/*mins* $< x <$ 40 min/*mins*

(iii) 15 min/*mins* $< x <$ 45 min/*mins*

(d) $2.4 = \mu_1 - 2\sigma$

$2\sigma = 3.2 - 2.4$

$= 0.8$

$\therefore P(X < 2.4) = P(X > \mu_2)$

$\therefore \mu_2 = 3.2 + 2\sigma$

$= 4.0$

18 (a) $Z = \dfrac{20 - 24}{\sqrt{3.7}}$

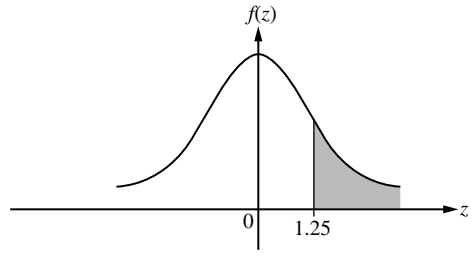
$= -2.0795$

(b) $1.319 = \dfrac{11.5 - 9.6}{\sigma}$

$$\sigma = \frac{11.5 - 9.6}{1.319}$$

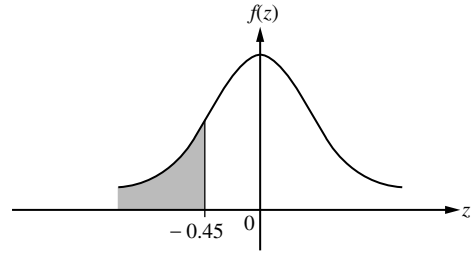
$= 1.44$

19 (a) $P(Z > 1.25) = 0.1056$



(b) $P(Z < -0.45) = P(Z > 0.45)$

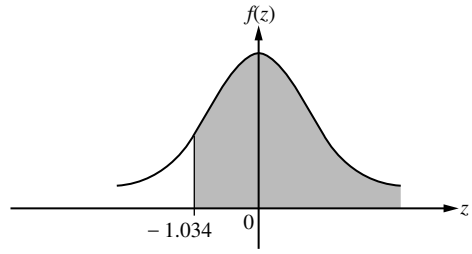
$= 0.3246$



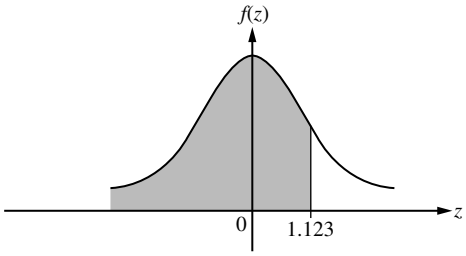
(c) $P(Z > -1.034) = 1 - P(Z > 1.034)$

$= 1 - 0.1506$

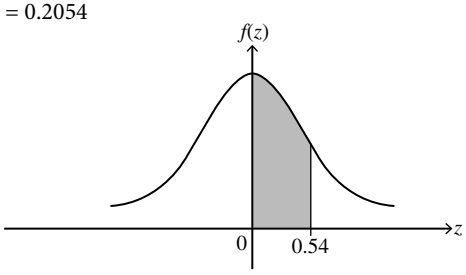
$= 0.8494$



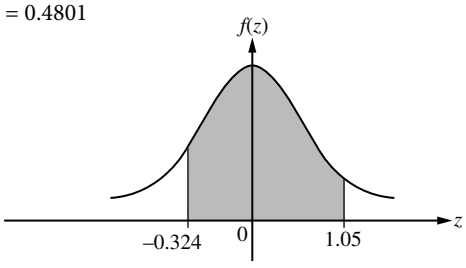
(d) $P(Z < 1.123) = 1 - P(Z > 1.123)$
$= 1 - 0.1308$
$= 0.8692$



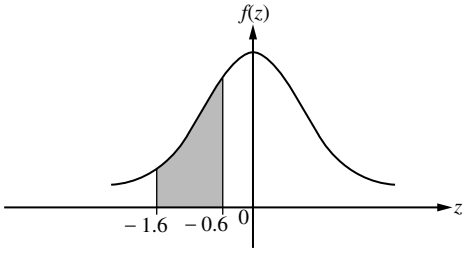
(e) $P(0 < Z < 0.54) = 0.5 - P(Z > 0.54)$
$= 0.5 - 0.2946$
$= 0.2054$



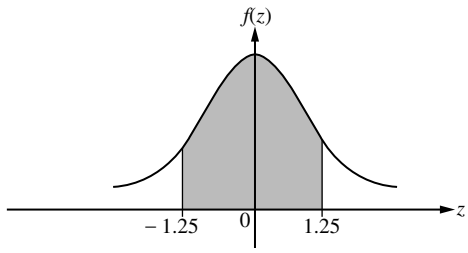
(f) $P(-0.324 < Z < 1.05)$
$= 1 - P(Z > 1.05) - P(Z > 0.324)$
$= 1 - 0.1469 - 0.373$
$= 0.4801$



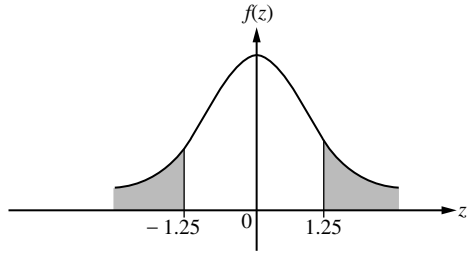
(g) $P(-1.6 < Z < -0.6)$
$= P(Z > 0.6) - P(Z > 1.6)$
$= 0.2743 - 0.0548$
$= 0.2195$



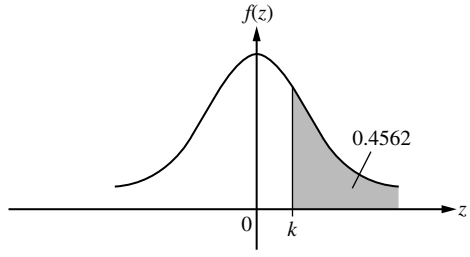
(h) $P|Z| < 1.25)$
$= P(-1.25 < Z < 1.25)$
$= 1 - 2(0.1057)$
$= 0.7886$



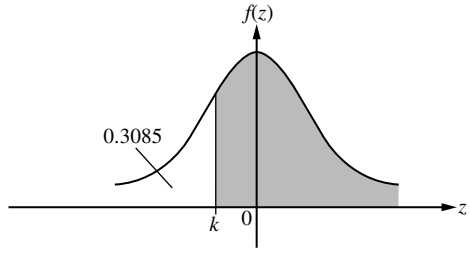
(i) $P(|Z| > 1.25)$
$= P(Z < -1.25) + P(Z > 1.25)$
$= 2\ (0.1056)$
$= 0.2112$



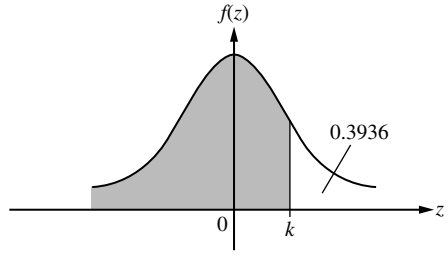
20 (a) $P(Z > k) = 0.4562$
$k = 0.11$



(b) $P(Z > k) = 0.6915$
$P(Z < k) = 1 - 0.6915$
$= 0.3085$
$k = -0.5$



(c) $P(Z < k) = 0.6064$
$P(Z > k) = 1 - 0.6064$
$= 0.3936$
$k = 0.27$



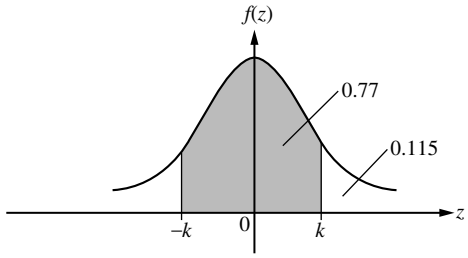
(d) $P(|Z| > k) = 0.77$

$P(-k < Z < k) = 0.77$

$P(Z > k) = \dfrac{1 - 0.77}{2}$

$= 0.115$

$k = 1.2$



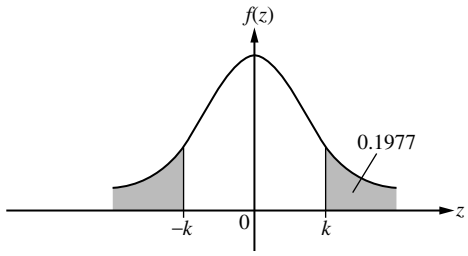
(e) $P(|Z| > k) = 0.3953$

$P(Z > k) = \dfrac{0.3952}{2}$

$= 0.19765$

≈ 0.1977

$k = 0.85$



21 (a) $\mu = 4.6$

(b) $P(X < a) = \dfrac{1 - 0.95}{2}$ (guna hukum/*use the rule of* 68-95-99.8)

$= 0.025$

$P(X > a) = 0.975$

atau/*or*

$P(Z > -2) = 1 - 0.0228$ (2 sisihan piawai daripada min/2 *standard deviation from mean*)

$= 0.9772$

(c) $b = 4.6 + 3\sigma$

$= 4.6 + 3\sqrt{0.25}$

$= 6.1$

22 Berdasarkan hukum 68-95-99.8,
Based on the 69-95-99.8 *rule,*

$\mu - 2\sigma = 8$

$\mu + 2\sigma = 12$

$2\mu = 8 + 12$

$\mu = 10$

$2\sigma = 12 - 10$

$\sigma = 1$

23 $X \sim N(500, 85^2)$

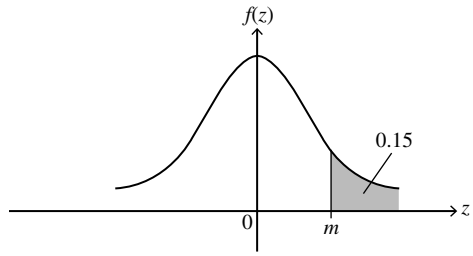
Li Lian mesti berada di kumpulan 15% teratas/*Li Lian must be in the top* 15%

$P(X > m) = 0.15$

$P\left(Z > \dfrac{m - 500}{85}\right) = 0.15$

$\dfrac{m - 500}{85} = 1.036$

$m = 588.06$



Markah minimum ialah 588.06. Markah Li Lian rendah daripada markah minimum, maka dia gagal diterima oleh universiti.

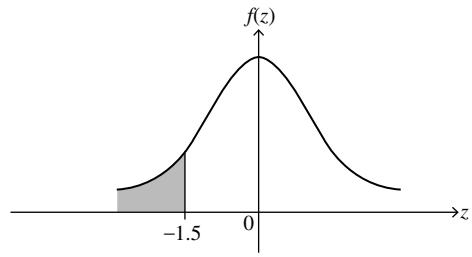
The minimum mark is 588.06. *Li Lian's mark is lower than the minimum mark, therefore she will not be admitted to this university.*

24 $X \sim N(12, 2^2)$

(a) $P(X < 9) = P\left(Z < \dfrac{9 - 12}{2}\right)$

$= P(Z < -1.5)$

$= 0.0668$

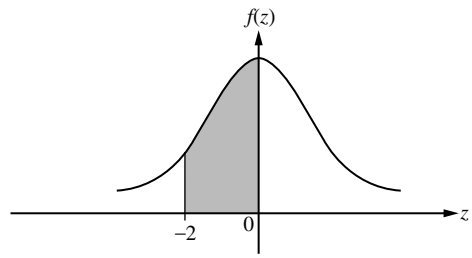


(b) $P(8 < X < 12 = P\left(\dfrac{8 - 12}{2} < Z < \dfrac{12 - 12}{2}\right)$

$= P(-2 < Z < 0)$

$= 0.5 - 0.0228$

$= 0.4772$



Praktis Sumatif

Kertas 1

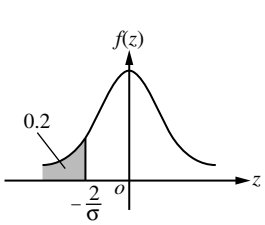
1 $X \sim N(\mu, \sigma^2)$

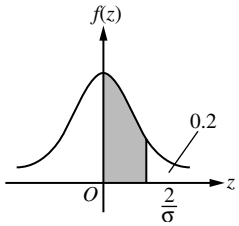
$P\left(X < \dfrac{X_1 - \mu}{\sigma}\right) = 0.2$

$P\left(Z < \dfrac{-2}{\sigma}\right) = 0.2$

$P\left(\mu < X < \dfrac{X_2 - \mu}{\sigma}\right) = 0.3$

$P\left(0 < Z < \dfrac{2}{\sigma}\right) = 0.3$





(a) $\frac{2}{\sigma} = 0.842$

$\sigma = \frac{2}{0.842}$

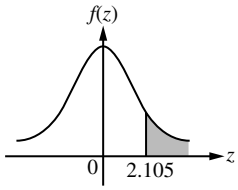
$= 2.375$

(b) $P\left(Z > \frac{5}{2.375}\right) = P(Z > 2.105)$

$= 0.0177$

$= 0.018$

(2 a.b./2 *s.f.*)



2 $p = \frac{5}{7}$, $n = 10$

$P(X = 10) = {}^{10}C_{10}\left(\frac{5}{7}\right)^{10}\left(\frac{2}{7}\right)^{0}$

$= 0.03457$

$n = 270$,

bilangan jangkaan jurujual/*expected number of salespeople*

$= np$

$= 270\ (0.03457)$

$= 9.3339$

≈ 9

Jumlah ganjaran wang tunai/*The total cash reward*

$= 9 \times$ RM2 000

= RM18 000

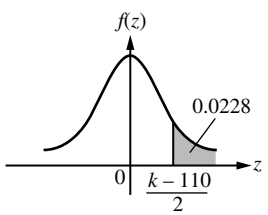
Peruntukan sebanyak RM16 000 itu tidak mencukupi kerana dijangka 9 orang jurujual yang dapat mencapai target jualan.

The allocation of RM16 000 *is not sufficient because* 9 *salespeople are expected to achieve the sales target.*

3 $t \sim$ N(110 min, 4 min^2)

$P(t \geqslant k) = 0.0228$

$P\left(Z > \frac{k - 110}{2}\right) = 0.0228$



$\frac{k - 110}{2} = 2$

$k = 2(2) + 110$

$= 114$ minit/*minutes*

Jumlah tempoh masa asal sehingga waktu perlepasan kapal terbang *N*

Total scheduled duration until the departure time of airplane N

$= 110 + 65$

$= 175$ minit/*minutes*

Tempoh masa sebenar perjalanan Sam

Sam's actual travelling duration

$= 114 + 55$

$= 169$ minit/*minutes*

Sam masih sempat menaiki kapal terbang *N* kerana tempoh masa digunakan adalah kurang daripada tempoh masa yang dijadualkan.

Sam was able to catch the airplane N because the time used is less than the scheduled duration.

4 (a) $\mu = 0$

$\sigma = 1$

(b) $P(Z > 1) = 0.1578$

$\therefore\ P(Z < 1) = 1 - 0\ 1578$

$= 0.8422$

atau/*or*

$P(0 < Z < 1) = \frac{0.68}{2}$ (Hukum 68-95-99.8 *rule*)

$\therefore\ P(Z < 1) = 0.5 + 0\ 34$

$= 0.84$

5 (a) $P(X = 3) = 0.0244$

(b) $P(X \leqslant 2) = 0.4437 + 0.3915 + 0.1382$

$= 0.9734$

6 $X \sim$ B(8, 0.08)

(a) $P(X = 0) = {}^{8}C_{0}(0.08)^{0}(0.92)^{8}$

$= 0.5132$

(b) $P(X < 2) = P(X = 0) + P(X = 1)$

$= {}^{8}C_{0}(0.08)^{0}(0.92)^{8} + {}^{8}C_{1}(0.08)^{1}(0.92)^{7}$

$= 0.5132 + 0.3570$

$= 0.8702$

7 (a) $4k = 1 - (0.2373 + 0.3955 + 0.0146 + 0.0010)$

$k = \frac{1 - 0.6484}{4}$

$= 0.0879$

(b) $P(X = 0) = 0.2373$

${}^{5}C_{0}(p)^{0}(1 - p)^{5} = 0.2373$

$1 - p = \sqrt[5]{0.2373}$

$p = 0.25$

8 $X \sim$ B (20, 0.68)

(a) Nilai jangkaan/*Expected number*, $\mu = np$

$= 20(0.68)$

$= 13.6$

(b) Sisihan piawai/*Standard deviation*,

$\sigma = \sqrt{npq}$

$= \sqrt{20(0.68)(0.32)}$

$= 4.352$

Kertas 2

1 (a) $X = \{x: x \in \mathbb{Z}, 0 \leqslant x \leqslant 20\}$
(atau senarai 0 hingga 20/*or list from* 0 *to* 20)

(b) X merupakan pemboleh ubah rawak yang bertaburan binomial kerana skesudahan setiap soalan ada 2 sahaja, iaitu betul atau salah. Kebarangkalian untuk menjawab soalan dengan betul adalah sama untuk setiap soalan dan tidak bergantung kepada kesudahan yang lepas.
X is a random variable that is binomially distributed because each question has only 2 *outcomes, which is either true or false. The probability to answer each question correctly is the same and it does not depend on the previous outcome.*

(c) $\mu = np$
$= 20\left(\frac{1}{4}\right)$
$= 5$
$\sigma = \sqrt{npq}$
$= \sqrt{20\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)}$
$= 3.75$

(d) $X \sim \mathrm{B}\left(10, \frac{1}{4}\right)$

Bilangan soalan minimum yang mesti dijawab dengan betul
The minimum number of questions must be answered correctly
$= 80\% \times 20$
$= 16$

$$P(X \geqslant 6) = {}^{10}C_6\left(\frac{1}{4}\right)^6\left(\frac{3}{4}\right)^4 + {}^{10}C_7\left(\frac{1}{4}\right)^7\left(\frac{3}{4}\right)^3 + {}^{10}C_8\left(\frac{1}{4}\right)^8\left(\frac{3}{4}\right)^2 + {}^{10}C_9\left(\frac{1}{4}\right)^9\left(\frac{3}{4}\right)^1 + {}^{10}C_{10}\left(\frac{1}{4}\right)^{10}\left(\frac{3}{4}\right)^0$$
$$= 0.016222 + 0.00309 + 0.000386 + 0.00002861 + 0.000000953$$
$$= 0.01973$$

2 (a) p = kebarangkalian memenangi satu patung beruang/*probability of winning win a teddy bear*
$= 0.8^3$
$= 0.512$
RM20 $\rightarrow$ 4 percubaan/*trials*
$\therefore n = 4$
$P(X \geqslant 1) = 1 - P(X = 0)$
$= 1 - {}^4C_0\,(0.512)^0(0.488)^4$
$= 0.9434$

May patut bermain kerana dia mempunyai 94.34% peluang untuk memenangi sekurang-kurangnya satu patung beruang.
May should play the game as she has 94.34% *chances to win at least a teddy bear.*

(b) Bilangan jangkaan bagi percubaan yang berjaya = min
The expected number of successful trials = mean
$np = 4$
$n(0.512) = 4$
$n = 7.8$
≈ 8

Perbelanjaan minimum/*The minimum spending*
$= 8 \times$ RM5
$=$ RM40