Penyelesaian Lengkap

Praktis 2

Praktis Formatif

1 (a) $\underset{x\to 1}{\text{had}}(x-1)/\underset{x\to 1}{\lim}(x-1) = 1-1$
$= 0$

(b) $\underset{x\to 2}{\text{had}}\dfrac{x^2-2}{x}/\underset{x\to 2}{\lim}\dfrac{x^2-2}{x} = \dfrac{2^2-2}{2}$
$= 1$

(c) $\underset{x\to 0}{\text{had}}\dfrac{2x-5}{x+3}/\underset{x\to 0}{\lim}\dfrac{2x-5}{x+3} = \dfrac{2(0)-5}{(0)+3}$
$= -\dfrac{5}{3}$

(d) $\underset{x\to a}{\text{had}}(x^2-2ax)/\underset{x\to a}{\lim}(x^2-2ax) = a^2-2a(a)$
$= -a^2$

2 (a) $\underset{x\to -2}{\text{had}}\dfrac{\frac{7}{x}+\frac{7}{2}}{x+2}/\underset{x\to -2}{\lim}\dfrac{\frac{7}{x}+\frac{7}{2}}{x+2}$
$= \underset{x\to -2}{\text{had}}\dfrac{14+7x}{2x(x+2)}/\underset{x\to -2}{\lim}\dfrac{14+7x}{2x(x+2)}$
$= \dfrac{7}{2(-2)}$
$= -\dfrac{7}{4}$

(b) $\underset{x\to 2}{\text{had}}\dfrac{x^2-4}{x-2}/\underset{x\to 2}{\lim}\dfrac{x^2-4}{x-2}$
$= \underset{x\to 2}{\text{had}}\dfrac{(x+2)(x-2)}{x-2}/\underset{x\to 2}{\lim}\dfrac{(x+2)(x-2)}{x-2}$
$= 2+2$
$= 4$

(c) $\underset{x\to 5}{\text{had}}\dfrac{x^2+4x-45}{x-5}/\underset{x\to 5}{\lim}\dfrac{x^2+4x-45}{x-5}$
$= \underset{x\to 5}{\text{had}}\dfrac{(x-5)(x+9)}{x-5}/\underset{x\to 5}{\lim}\dfrac{(x-5)(x+9)}{x-5}$
$= \underset{x\to 5}{\text{had}}(x+9)/\underset{x\to 5}{\lim}(x+9)$
$= 5+9$
$= 14$

3 (a) $\underset{x\to 9}{\text{had}}\dfrac{\sqrt{x}-3}{x-9}/\underset{x\to 9}{\lim}\dfrac{\sqrt{x}-3}{x-9}$
$= \underset{x\to 9}{\text{had}}\dfrac{(\sqrt{x}-3)(\sqrt{x}+3)}{(x-9)(\sqrt{x}+3)}/\underset{x\to 9}{\lim}\dfrac{(\sqrt{x}-3)(\sqrt{x}+3)}{(x-9)(\sqrt{x}+3)}$
$= \underset{x\to 9}{\text{had}}\dfrac{(x-9)}{(x-9)(\sqrt{x}+3)}/\underset{x\to 9}{\lim}\dfrac{(x-9)}{(x-9)(\sqrt{x}+3)}$
$= \underset{x\to 9}{\text{had}}\dfrac{1}{(\sqrt{x}+3)}/\underset{x\to 9}{\lim}\dfrac{1}{(\sqrt{x}+3)}$
$= \dfrac{1}{6}$

(b) $\underset{x\to 2}{\text{had}}\dfrac{\sqrt{6-x}-2}{3-\sqrt{11-x}}/\underset{x\to 2}{\lim}\dfrac{\sqrt{6-x}-2}{3-\sqrt{11-x}}$
$= \underset{x\to 2}{\text{had}}\dfrac{(\sqrt{6-x}-2)(3+\sqrt{11-x})}{(3-\sqrt{11-x})(3+\sqrt{11-x})}$
$\underset{x\to 2}{\lim}\dfrac{(\sqrt{6-x}-2)(3+\sqrt{11-x})}{(3-\sqrt{11-x})(3+\sqrt{11-x})}$
$= \underset{x\to 2}{\text{had}}\dfrac{(\sqrt{6-x}-2)(3+\sqrt{11-x})}{9-(11-x)}$
$\underset{x\to 2}{\lim}\dfrac{(\sqrt{6-x}-2)(3+\sqrt{11-x})}{9-(11-x)}$
$= \underset{x\to 2}{\text{had}}\dfrac{(\sqrt{6-x}-2)(3+\sqrt{11-x})(\sqrt{6-x}+2)}{(x-2)(\sqrt{6-x}+2)}$
$\underset{x\to 2}{\lim}\dfrac{(\sqrt{6-x}-2)(3+\sqrt{11-x})(\sqrt{6-x}+2)}{(x-2)(\sqrt{6-x}+2)}$
$= \underset{x\to 2}{\text{had}}\dfrac{(6-x-4)(3+\sqrt{11-x})}{(x-2)(\sqrt{6-x}+2)}$
$\underset{x\to 2}{\lim}\dfrac{(6-x-4)(3+\sqrt{11-x})}{(x-2)(\sqrt{6-x}+2)}$
$= \underset{x\to 2}{\text{had}}\dfrac{-(x-2)(3+\sqrt{11-x})}{(x-2)(\sqrt{6-x}+2)}/\underset{x\to 2}{\lim}\dfrac{-(x-2)(3+\sqrt{11-x})}{(x-2)(\sqrt{6-x}+2)}$
$= \underset{x\to 2}{\text{had}}\dfrac{-(3+\sqrt{11-x})}{(\sqrt{6-x}+2)}/\underset{x\to 2}{\lim}\dfrac{-(3+\sqrt{11-x})}{(\sqrt{6-x}+2)}$
$= \dfrac{-(3+\sqrt{11-2})}{(\sqrt{6-2}+2)}$
$= \dfrac{-6}{4}$
$= -\dfrac{3}{2}$

4 (a) $f(4) = 3$

(b) $\underset{x\to 4}{\text{had}}f(x)/\underset{x\to 4}{\lim}f(x)$ = tidak wujud/*does not exist*

Had kiri dan had kanan bagi fungsi $f(x)$ adalah tidak sama apabila x menghampiri 4.
The left-hand limit and the right-hand limit of $f(x)$ *are different as* x *approaches* 4.

(c) $\underset{x\to 3}{\text{had}}f(x)/\underset{x\to 3}{\lim}f(x) = 3$

5 (a)
$y = 3x+5$
$y+\delta y = 3(x+\delta x)+5$
$= 3x+3\delta x+5$
$\delta y = 3\delta x$
$\dfrac{\delta y}{\delta x} = 3$

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$\frac{dy}{dx} \approx \underset{\delta x \to 0}{\text{had}}\ 3 / \lim_{\delta x \to 0} 3$

$\frac{dy}{dx} = 3$

(b) $y = -x^3$

$y + \delta y = -(x + \delta x)^3$

$= -[x^3 + 3x^2 \delta x + 3x(\delta x)^2 + (\delta x)^3]$

$\delta y = -[3x^2 \delta x + 3x(\delta x)^2 + (\delta x)^3]$

$\frac{\delta y}{\delta x} = -[3x^2 + 3x\,\delta x + (\delta x)^2]$

$\frac{dy}{dx} \approx \underset{\delta x \to 0}{\text{had}} - [3x^2 + 3x\,\delta x + (\delta x)^2] /$

$\lim_{\delta x \to 0} - [3x^2 + 3x\,\delta x + (\delta x)^2]$

$\frac{dy}{dx} = -3x^2$

(c) $y = \frac{5}{x^2}$

$y + \delta y = \frac{5}{(x + \delta x)^2}$

$\delta y = \frac{5}{x^2 + 2x\,\delta x + (\delta x)^2} - \frac{5}{x^2}$

$= \frac{5x^2 - 5x^2 - 10x\,\delta x - 5(\delta x)^2}{x^2[x^2 + 2x\,\delta x + (\delta x)^2]}$

$= \frac{-10x\,\delta x - 5(\delta x)^2}{x^2[x^2 + 2x\,\delta x + (\delta x)^2]}$

$\frac{\delta y}{\delta x} = \frac{-10x - 5\delta x}{x^2[x^2 + 2x\,\delta x + (\delta x)^2]}$

$\frac{dy}{dx} \approx \underset{\delta x \to 0}{\text{had}} \frac{-10x - 5\delta x}{x^2[x^2 + 2x\,\delta x + (\delta x)^2]}$

$\lim_{\delta x \to 0} \frac{-10x - 5\delta x}{x^2[x^2 + 2x\,\delta x + (\delta x)^2]}$

$\frac{dy}{dx} = \frac{-10x}{x^2(x)^2} = \frac{-10}{x^3}$

6 (a) $y = x^2 - ax + b$

$y + \delta y = (x + \delta x)^2 - a(x + \delta x) + b$

$= x^2 + 2x\delta x + (\delta x)^2 - ax - a\delta x + b$

$\delta y = 2x\delta x + (\delta x)^2 - a\delta x$

$\frac{\delta y}{\delta x} = 2x + \delta x - a$

$\frac{dy}{dx} \approx \underset{\delta x \to 0}{\text{had}}\ 2x + \delta x - a / \lim_{\delta x \to 0} 2x + \delta x - a$

$\frac{dy}{dx} = 2x - a$

(b) $\frac{dy}{dx} = 2$ pada/*at* $(2, -3)$

$2(2) - a = 2$

$a = 2$

Gantikan $(2, -3)$ ke dalam $y = x^2 - 2x + b$,

Substitute $(2, -3)$ *into* $y = x^2 - 2x + b$,

$-3 = (2)^2 - 2(2) + b$

$b = -3$

7 (a) $y = -x^{\frac{4}{3}}$

$\frac{dy}{dx} = -\frac{4x^{\frac{1}{3}}}{3}$

$= -\frac{4}{3}x^{\frac{1}{3}}$

(b) $y = -2x^{-2}$

$\frac{dy}{dx} = 4x^{-3}$

8 (a) $\frac{d}{dx}(2x^2 + 3x - 9) = 4x + 3$

(b) $\frac{d}{dx}(x^2 + \frac{2}{x}) = \frac{d}{dx}(x^2 + 2x^{-1})$

$= 2x - 2x^{-2}$

$= 2x - \frac{2}{x^2}$

(c) $\frac{d}{dx}\left(5x^3 + 2x^2 + 4x - 7 - \frac{1}{x} + \frac{3}{x^2}\right)$

$= \frac{d}{dx}(5x^3 + 2x^2 + 4x - 7 - x^{-1} + 3x^{-2})$

$= 15x^2 + 4x + 4 + x^{-2} - 6x^{-3}$

$= 15x^2 + 4x + 4 + \frac{1}{x^2} - \frac{6}{x^3}$

9 (a) $f(x) = x^3 + 3x^2 - 5x - 15$

$f'(x) = 3x^2 + 6x - 5$

(b) $f(x) = x^2 - 1 + 4x^{-1}$

$f'(x) = 2x - 4x^{-2}$

(c) $f(x) = \frac{(x - 2)(x + 1)}{(x - 2)}$

$= x + 1$

$f'(x) = 1$

10 (a) $f'(x) = 7(2)(1 + 4x)^6(4)$

$= 56(1 - 4x)^6$

(b) $f(x) = 2(5x^2 - 3x)^{-10}$

$f'(x) = -20(5x^2 - 3x)^{-11}(10x - 3)$

$= \frac{-20(10x - 3)}{(5x^2 - 3x)^{11}}$

11 (a) $y = 6x^2[x(1 + 5x)]^3$

$= 6x^2(x^3)(1 + 5x)^3$

$= 6x^5(1 + 5x)^3$

$\frac{dy}{dx} = (1 + 5x)^3 \frac{d}{dx}(6x^5) + (6x^5)\frac{d}{dx}(1 + 5x)^3$

$= (1 + 5x)^3(30x^4) + (6x^5)(3)(1 + 5x)^2(5)$

$= 30x^4(1 + 5x)^2[1 + 5x + 3x]$

$= 30x^4(1 + 5x)^2(1 + 8x)$

(b) $\frac{dy}{dx} = (1 - 2x^2)^{10}\frac{d}{dx}(4x^2 - 3x) + (4x^2 - 3x)\frac{d}{dx}(1 - 2x^2)^{10}$

$= (1 - 2x^2)^{10}(8x - 3) + (4x^2 - 3x)(10)(1 - 2x^2)^9(-4x)$

$= (1 - 2x^2)^9[(1 - 2x^2)(8x - 3) - 40x(4x^2 - 3x)]$

$= (1 - 2x^2)^9[8x - 3 - 16x^3 + 6x^2 - 160x^3 + 120x^2]$

$= (1 - 2x^2)^9(8x - 3 - 176x^3 + 126x^2)$

12 (a) $\frac{dy}{dx} = \frac{(2x + 1)\frac{d}{dx}(x - 2) - (x - 2)\frac{d}{dx}(2x + 1)}{(2x + 1)^2}$

$= \frac{(2x + 1)(1) - (x - 2)(2)}{(2x + 1)^2}$

$= \frac{2x + 1 - 2x + 4}{(2x + 1)^2}$

$= \frac{5}{(2x + 1)^2}$

(b) $\frac{dy}{dx} = \frac{(2x - 1)^2\frac{d}{dx}(x^3) - (x^3)\frac{d}{dx}(2x - 1)^2}{[(2x - 1)^2]^2}$

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$= \dfrac{(2x-1)^2(3x^2) - (x^3)(2)(2x-1)(2)}{(2x-1)^4}$

$= \dfrac{x^2(2x-1)[3(2x-1) - (4x)]}{(2x-1)^4}$

$= \dfrac{x^2(6x-3-4x)}{(2x-1)^3}$

$= \dfrac{x^2(2x-3)}{(2x-1)^3}$

13 $y = \dfrac{2x-3}{x^2+1}$

$x^2 + 1 > 0,\ 2x - 3 > 0$

$x > \dfrac{3}{2}$

$\dfrac{dy}{dx} = \dfrac{(x^2+1)(2) - (2x-3)(2x)}{(x^2+1)^2}$

$= \dfrac{2x^2 + 2 - 4x^2 + 6x}{(x^2+1)^2}$

$= \dfrac{2 + 6x - 2x^2}{(x^2+1)^2}$

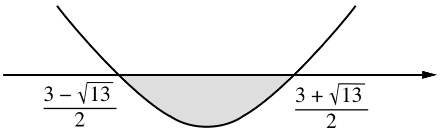
$= \dfrac{-2(x^2 - 3x - 1)}{(x^2+1)^2}$

$\dfrac{dy}{dx} > 0 \qquad (x^2+1) > 0,\ -2(x^2 - 3x - 1) > 0$

$x^2 - 3x - 1 < 0$

$x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)}$

$= \dfrac{3 \pm \sqrt{13}}{2}$



$\dfrac{3-\sqrt{13}}{2} < x < \dfrac{3+\sqrt{13}}{2}$

$\therefore \dfrac{3}{2} < x < \dfrac{3+\sqrt{13}}{2}$

14 $y = 5(x-4) - 2(x-4)^2$

$= 5x - 20 - 2(x-4)^2$

$\dfrac{dy}{dx} = 5 - 2(2)(x-4)(1)$

$= 5 - 4(x-4)$

$x = 2,\ \dfrac{dy}{dx} = 5 - 4(2-4)$

$= 13$

15 $\dfrac{d}{dx}(ax^m + bx^n) = 12x^s + 9x^t$

$am(x^{m-1}) + bn(x^{n-1}) = 12x^s + 9x^t$

$am = 12,\ bn = 9,\ s = m-1,\ t = n-1$

(a) $\dfrac{s}{t} = \dfrac{m-1}{n-1}$

$= \dfrac{\frac{12}{a} - 1}{\frac{9}{b} - 1}$

$= \dfrac{\frac{12-a}{a}}{\frac{9-b}{b}}$

$= \dfrac{b(12-a)}{a(9-b)}$

(b) $\dfrac{s}{t} = \dfrac{5}{3}$ $\qquad$ $\dfrac{m}{n} = \dfrac{3}{2}$

$\dfrac{b(12-a)}{a(9-b)} = \dfrac{5}{3}$ $\qquad$ $2m = 3n$

$36b - 3ab = 45a - 5ab$ $\qquad$ $2\left(\dfrac{12}{a}\right) = 3\left(\dfrac{9}{b}\right)$

$36b + 2ab = 45a \ldots ①$ $\qquad$ $b = \dfrac{9}{8}a \ldots ②$

Gantikan ② ke dalam ①/*Substitute* ② *into* ①,

$36\left(\dfrac{9}{8}a\right) + 2a\left(\dfrac{9}{8}a\right) = 45a$

$36a + 2a^2 - 40a = 0$

$2a^2 - 4a = 0$

$2a(a-2) = 0$

$a = 2 (a > 0)$

$b = \dfrac{9}{8}(2)$

$= \dfrac{9}{4}$

(c) $m = \dfrac{12}{2}$ $\qquad$ $s = 6 - 1$

$= 6$ $\qquad$ $= 5$

$n = 9 \div \dfrac{9}{4}$ $\qquad$ $t = 4 - 1$

$= 4$ $\qquad$ $= 3$

16 (a) $\dfrac{dy}{dx} = 12x^2 - 7x^{-2}$

$\dfrac{d^2y}{dx^2} = 24x + 14x^{-3}$

(b) $\dfrac{dy}{dx} = 3(-2)(x^2+1)^{-3}(2x) = \dfrac{-12x}{(x^2+1)^3}$

$\dfrac{d^2y}{dx^2} = \dfrac{(x^2+1)^3 \frac{d}{dx}(-12x) - (-12x)\frac{d}{dx}(x^2+1)^3}{[(x^2+1)^3]^2}$

$= \dfrac{(x^2+1)^3(-12) - (-12x)(3)(x^2+1)^2(2x)}{(x^2+1)^6}$

$= \dfrac{(-12)(x^2+1)^2[x^2 + 1 - 6x^2]}{(x^2+1)^6}$

$= \dfrac{-12(1 - 5x^2)}{(x^2+1)^4}$

17 $\dfrac{d}{dx}\left(\dfrac{x^4}{12} - x^3 + \dfrac{9}{2}x^2 + 6x - 3\right) = \dfrac{x^3}{3} - 3x^2 + 9x + 6$

$\dfrac{d}{dx}\left(\dfrac{x^3}{3} - 3x^2 + 9x + 6\right) = x^2 - 6x + 9$

$\dfrac{d^2}{dx^2}\left(\dfrac{x^4}{12} - x^3 + \dfrac{9}{2}x^2 + 6x - 3\right)$

$= x^2 - 6x + 9$

$= (x-3)^2$

Terbukti bahawa sentiasa positif bagi semua nilai x

Proven that always positive for all the values of x

18 $f'(x) = 2x^3 + 3px^2 + 3x - 16,$ $\qquad$ $f''(x) = 0$

$6x^2 + 6px + 3 = 0$

$2x^2 + 2px + 1 = 0$

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Sekurang-kurangnya satu penyelesaian nyata
At least one real solution

$$\begin{aligned} b^2 - 4ac &\geqslant 0 \\ (2p)^2 - 4(2)(1) &\geqslant 0 \\ 4p^2 - 8 &\geqslant 0 \\ p^2 - 2 &\geqslant 0 \\ (p + \sqrt{2})(p - \sqrt{2}) &\geqslant 0 \end{aligned}$$

$\therefore p \leqslant -\sqrt{2}$ atau/*or* $p \geqslant \sqrt{2}$

19 (a) $m_t = \dfrac{dy}{dx}$

$= 4 + 8x^{-2}$

$m_{t_{x=4}} = 4 + \dfrac{8}{4^2}$

$= 4.5$

(b) $m_t = \dfrac{(3 - 2x)\frac{d}{dx}(4 - 3x^2) - (4 - 3x^2)\frac{d}{dx}(3 - 2x)}{(3 - 2x)^2}$

$= \dfrac{(3 - 2x)(-6x) - (4 - 3x^2)(-2)}{(3 - 2x)^2}$

$m_{t_{x=2}} = \dfrac{[(3 - 2(2)](-6)(2) - (4 - 3(2)^2)(-2)}{[3 - 2(2)]^2}$

$= -4$

20 (a) $m_t = \dfrac{dy}{dx}$

$= 6x^2 - 6x$

$m_{t_{x=1}} = 6(1)^2 - 6(1)$

$= 0$

(b) $m_t = \dfrac{dy}{dx}$

$$\begin{aligned} 8 &= x^2 + 2x \\ x^2 + 2x - 8 &= 0 \\ (x + 4)(x - 2) &= 0 \\ x &= -4, 2 \end{aligned}$$

$y = \dfrac{(-4)^3}{3} + (-4)^2 - 1$ atau/*or* $y = \dfrac{2^3}{3} + 2^2 - 1$

$= -\dfrac{19}{3}$ $\qquad = \dfrac{17}{3}$

Koordinat/*Coordinates*: $\left(-4, -\dfrac{19}{3}\right), \left(2, \dfrac{17}{3}\right)$

(c) $\dfrac{dy}{dx} = 2ax + b$

$m_{t_{x=2}} = 5$ $\qquad m_{t_{x=-3}} = 0$

$4a + b = 5 \ldots$ ① $\qquad -6a + b = 0 \ldots$ ②

① – ②: $10a = 5$

$a = \dfrac{1}{2}$

$b = 5 - 4\left(\dfrac{1}{2}\right)$

$= 3$

21 (a) (i) $m_t = \dfrac{dy}{dx} = 1 + 8x^{-2}$

$x = 2, y = 2 + 2 - \dfrac{8}{2} = 0$

Pada titik (2, 0)/*At point* (2, 0),

$m_t = 1 + \dfrac{8}{2^2} = 3$

$m_n = -\dfrac{1}{3}$

Persamaan tangen/*Equation of tangent*:

$y - 0 = 3(x - 2)$

$y = 3x - 6$

Persamaan normal/*Equation of normal*:

$$\begin{aligned} y - 0 &= -\frac{1}{3}(x - 2) \\ 3y &= -x + 2 \\ x + 3y - 2 &= 0 \end{aligned}$$

(ii) $y = 2 + x - \dfrac{8}{x} \ldots$ ①

$x + 3y - 2 = 0 \ldots$ ②

Gantikan ① ke dalam ②/*Substitute* ① *into* ②,

$$\begin{aligned} x + 3\left(2 + x - \frac{8}{x}\right) - 2 &= 0 \\ x^2 + 6x + 3x^2 - 24 - 2x &= 0 \\ 4x^2 + 4x - 24 &= 0 \\ x^2 + x - 6 &= 0 \\ (x + 3)(x - 2) &= 0 \\ x &= -3, 2 \end{aligned}$$

$x = -3,\quad -3 + 3y - 2 = 0$

$y = \dfrac{5}{3}$

Normal menyilang lengkung itu sekali lagi pada titik $\left(-3, \dfrac{5}{3}\right)$.

The normal intersects the curve again at point $\left(-3, \dfrac{5}{3}\right)$.

(b) $m_t = \dfrac{dy}{dx}$

$= 2ax + b$

$m_{t_{x=4}} = 2a(4) + b$

$= 8a + b$

$y = ax^2 + bx$

Pada/*At* $P(4, 8)$,

$8 = a(4)^2 + b(4)$

$8 = 16a + 4b$

$4a + b = 2 \ldots$ ②

$m_{AB} = \dfrac{0 - 1}{12 - 4}$

$= -\dfrac{1}{8}$

$m_{t_{x=4}} \times m_{AB} = -1$

$8a + b = 8 \ldots$ ①

① – ②: $4a = 6$

$a = \dfrac{3}{2}$

Daripada/*From* ①: $b = 8 - 8\left(\dfrac{3}{2}\right) = -4$

22 (a) $m_t = 0$

$10x - 2 = 0$

$x = \dfrac{1}{5}$

$y = 5\left(\dfrac{1}{5}\right)^2 - 2\left(\dfrac{1}{5}\right) + 1 = \dfrac{4}{5}$

Titik pusingan/*Turning point* $= \left(\dfrac{1}{5}, \dfrac{4}{5}\right)$

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(i)

x	0	$\frac{1}{5}$	$\frac{2}{5}$
$\frac{dy}{dx}$	−2	0	2
Lakaran tangen *Sketch of the tangent*			
Lakaran graf *Sketch of the graph*			

$\therefore \left(\frac{1}{5}, \frac{4}{5}\right)$ ialah titik minimum/*is a minimum point*

(ii) $\frac{dy}{dx} = 10x - 2$

$\frac{d^2y}{dx^2} = 10 > 0$

$\therefore \left(\frac{1}{5}, \frac{4}{5}\right)$ ialah titik minimum/*is a minimum point*

(b) $y = \frac{x^2}{x+1}$

$$\frac{dy}{dx} = \frac{(x+1)\frac{d}{dx}(x^2) - (x^2)\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$= \frac{(x+1)(2x) - (x^2)(1)}{(x+1)^2}$$

$$= \frac{2x^2 + 2x - x^2}{(x+1)^2}$$

$$= \frac{x^2 + 2x}{(x+1)^2}$$

$$\frac{x^2 + 2x}{(x+1)^2} = 0$$

$x + 1 \neq 0$, $x(x + 2) = 0$

$x = 0$ atau/*or* $x = -2$

$y = \frac{(0)^2}{(0)+1} = 0$ $\qquad$ $y = \frac{(-2)^2}{(-2)+1} = -4$

Titik pusingan/*Turning points* = (0, 0), (−2, −4)

(i)

x	−3	−2	−1.5	−0.5	0	1
$\frac{dy}{dx}$	0.75	0	−3	−3	0	0.75
Lakaran tangen *Sketch of the tangent*						
Lakaran graf *Sketch of the graph*						

$\therefore$ (−2, −4) ialah titik maksimum
(−2, −4) *is a maximum point*
(0, 0) ialah titik minimum/*is a minimum point*

(ii) $\frac{dy}{dx} = \frac{x^2 + 2x}{(x+1)^2}$

$$\frac{d^2y}{dx^2} = \frac{(x+1)^2\frac{d}{dx}(x^2+2x) - (x^2+2x)\frac{d}{dx}(x+1)^2}{[(x+1)^2]^2}$$

$$= \frac{(x+1)^2(2x+2) - (x^2+2x)(2)(x+1)(1)}{(x+1)^4}$$

Apabila/*When* $x = 0$,

$$\frac{d^2y}{dx^2} = \frac{(0+1)^2(0+2) - (0+0)(2)(0+1)(1)}{(0+1)^4}$$

$$= 2 > 0$$

$\therefore$ (0, 0) ialah titik minimum/*is a minimum point*

Apabila/*When* $x = -2$,

$$\frac{d^2y}{dx^2} = \frac{(-2+1)^2(-4+2) - ((-2)^2 + 2(-2))(2)(-2+1)(1)}{(-2+1)^4} = -2 < 0$$

$\therefore$ (−2, −4) ialah titik maksimum
(−2, −4) *is a maximum point*

$$= -\frac{2}{3} < 0$$

(c) $y = 7 - x^3$

$\frac{dy}{dx} = -3x^2$

$-3x^2 = 0$

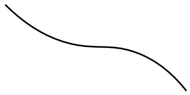
$x = 0$

$y = 7 - (0)^3$

$= 7$

Titik pusingan/*Turning point* = (0, 7)

(i)

x	− 1	0	1
$\frac{dy}{dx}$	− 3	0	− 3
Lakaran tangen *Sketch of the tangent*			
Lakaran graf *Sketch of the graph*			

$\therefore$ (0, 7) ialah titik lengkok balas
(0, 7) *is an inflection point*

(ii) $\frac{dy}{dx} = -3x^2$

$\frac{d^2y}{dx^2} = -6x$

Apabila/*When* $x = 0$, $\frac{d^2y}{dx^2} = 0$

$\therefore$ (0, 7) ialah titik lengkok balas
(0, 7) *is an inflection point*

23 (a) $r = x$

$V = \pi r^2 h$

$= \pi x^2(45 - 5x)$

$= 45\pi x^2 - 5\pi x^3$

V adalah maksimum apabila/*is maximum when*

$$\frac{dV}{dx} = 0$$

$90\pi x - 15\pi x^2 = 0$

$15\pi x(6 - x) = 0$

$15\pi x \neq 0$, $x = 6$

$$\frac{d^2V}{dx^2} = 90\pi - 30\pi x$$

$x = 6$, $\frac{d^2V}{dx^2} = 90\pi - 30\pi(6)$

$= -90\pi < 0$

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$x = 6$ menjadikan isi padu silinder yang dikeluarkan adalah maksimum.
$x = 6$ makes the volume of the cylinder taken out a maximum.

(b) $b = \sqrt{40 - h^2}$

$A = h(40 - h^2)^{\frac{1}{2}}$

Titik pegun wujud apabila
Stationary point exists when

$\frac{dA}{dx} = 0$

$(40 - h^2)^{\frac{1}{2}} \frac{d}{dh}(h) + (h)\frac{d}{dh}(40 - h^2)^{\frac{1}{2}} = 0$

$(40 - h^2)^{\frac{1}{2}} + \frac{h(-2h)}{2(40 - h^2)^{\frac{1}{2}}} = 0$

$\frac{40 - h^2 - h^2}{(40 - h^2)^{\frac{1}{2}}} = 0$

$(40 - h^2)^{\frac{1}{2}} \neq 0,\ 40 - 2h^2 = 0$

$2h^2 = 40$

$h = \sqrt{20}\ (h > 0)$

$b = \sqrt{40 - \sqrt{20}^2}$

$= \sqrt{20}$

$A = \sqrt{20} \times \sqrt{20} = 20$

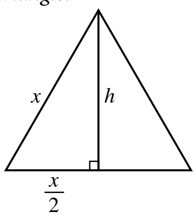
h	4	$\sqrt{20}$	5
$\frac{dA}{dx}$	1.63	0	−2.58
Lakaran tangen *Sketch of the tangent*			
Lakaran graf *Sketch of the graph*			

$\therefore A = 20$ ialah nilai maksimum/*is a maximum value*

(c) $3x + 4y = 120$

$y = \frac{120 - 3x}{4}$

Biar h mewakili tinggi segi tiga itu.
Let h represents the height of the triangle.



$h = \sqrt{x^2 - \left(\frac{x}{2}\right)^2}$

$= \frac{\sqrt{3}x}{2}$

$A = A_\Delta + A_\square$

$= \frac{1}{2}(x)\left(\frac{\sqrt{3}x}{2}\right) + y^2$

$= \frac{\sqrt{3}x^2}{4} + \left(\frac{120 - 3x}{4}\right)^2$

$= \frac{4\sqrt{3}x^2 + [3(40 - x)]^2}{16}$

$= \frac{4\sqrt{3}x^2 + 9(40 - x)^2}{16}$

$= \frac{9(40 - x)^2 + 4\sqrt{3}x^2}{16}$ (Tertunjuk/*Shown*)

A pegun apabila/*A is stationary when*

$\frac{dA}{dx} = 0$

$\frac{18(40 - x)(-1) + 8\sqrt{3}x}{16} = 0$

$\frac{18x - 720 + 8\sqrt{3}x}{16} = 0$

$18x - 720 + 8\sqrt{3}x = 0$

$(18 + 8\sqrt{3})x = 720$

$x = 22.6$

$\frac{d^2A}{dx^2} = \frac{18 + 8\sqrt{3}}{16}$

$= 1.991 > 0$

$\therefore A$ adalah minimum apabila/*A is minimum when* $x = 22.6$

24 (a) $\frac{dA}{dr} = 2\pi\left(-18r^{-2} + \frac{2r}{3}\right)$, $\frac{dA}{dt} = 2.1\pi \text{ cm}^2 \text{ s}^{-1}$

$r = 6$ cm, $\frac{dA}{dr} = 2\pi\left(-\frac{18}{(6)^2} + \frac{2(6)}{3}\right)$

$= 7\pi$

$\frac{dr}{dt} = \frac{dr}{dA} \times \frac{dA}{dt}$

$= \frac{1}{7\pi} \times (2.1\,\pi)$

$= 0.3 \text{ cm s}^{-1}$

(b) $\frac{dA}{dt} = 6 \text{ cm}^2 \text{ s}^{-1}$, $r = 5$ cm

(i) $A = 4\pi r^2$

$\frac{dA}{dr} = 8\pi r$

$\frac{dr}{dt} = \frac{dr}{dA} \times \frac{dA}{dt}$

$\frac{dr}{dt} = \frac{1}{8\pi(5)} \times 6$

$= \frac{3}{20\pi} \text{ cm s}^{-1}$

(ii) $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$\frac{dV}{dt} = \frac{dV}{dA} \times \frac{dA}{dt}$

$= \frac{dV}{dr} \times \frac{dr}{dA} \times \frac{dA}{dt}$

$\frac{dV}{dr} = 4\pi(5)^2 \times \frac{3}{20\pi}$

$= 15 \text{ cm}^3 \text{ s}^{-1}$

25 (a) $\frac{dL}{dm} = \frac{dL}{dx} \times \frac{dx}{dm}$

$= (4 - 2x) \times \left(\frac{1}{6}\right)$

$= \frac{2 - x}{3}$

$\delta L = 0.4$

$\frac{\delta L}{\delta m} \approx \frac{dL}{dm}$

$\delta m \approx \frac{dm}{dL} \times \delta L$

$= \frac{3}{2 - x} \times 0.4$

$x = 1, \delta m = 1.2$

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(b) $\frac{dy}{dx} = -\frac{36}{(2x-5)^3}$

(i) Apabila/*When* $x = 3$,

$\delta x = p$

$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$

$\delta y = \frac{-36}{[2(3)-5]^3} \times p$

$= -36p$

(ii) Apabila/*When* $y = 1$,

$\frac{9}{(2x-5)^2} = 1$

$(2x-5)^2 = 9$

$2x - 5 = \pm 3$

$x = 1, 4$

$\delta y = -p$

$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$

$\frac{-p}{\delta x} \approx \frac{-36}{[2(1)-5]^3}$ atau/*or* $\frac{-p}{\delta x} \approx \frac{-36}{[2(4)-5]^3}$

$\frac{-p}{\delta x} \approx \frac{4}{3}$ $\quad\frac{-p}{\delta x} \approx -\frac{4}{3}$

$\delta x \approx \pm\frac{3}{4}p$

(c) $\frac{dy}{dx} = 4x^3$

(i) $x = 2, \delta x = 0.03$

$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$

$\delta y \approx 4(2)^3 \times 0.03$

$= 0.96$

$y_n = y_o + \delta y$

$2.03^4 = 2^4 + 0.96$

$= 16.96$

(ii) $x = 2, \delta x = -0.01$

$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$

$\delta y \approx 4(2)^3 \times (-0.01)$

$= -0.32$

$y_n = y_o + \delta y$

$1.99^4 = 2^4 - 0.32$

$= 15.68$

Praktis Sumatif

Kertas 1

1 $\delta y = 4x\,\delta x + 2(\delta x)^2 + 3\delta x$

$\frac{\delta y}{\delta x} = 4x + 2\delta x + 3$

$\frac{dy}{dx} \approx \underset{\delta x \to 0}{\text{had}}\, 4x + 2\delta x + 3 / \lim_{\delta x \to 0} 4x + 2\delta x + 33$

$= 4x + 3$

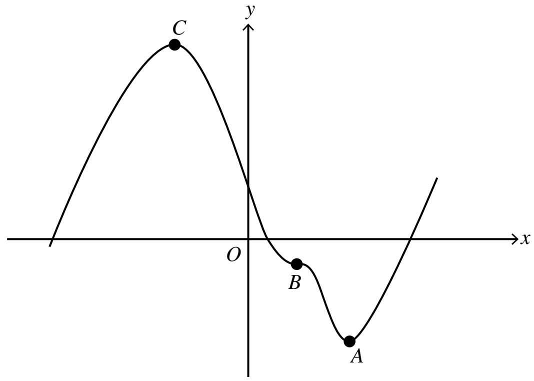
Apabila/*When* $x = 2$,

$\frac{dy}{dx} = 4(2) + 3 = 11$

2 $f'(x_a) = f'(x_b) = f'(x_c) = 0 \rightarrow A, B, C$ ialah titik pusingan
A, B, C are turning points

$f''(x_b) = 0 \rightarrow B$ ialah titik lengkok balas
B is an inflection point

$f''(x_c) < f''(x_b) < f''(x_a) \rightarrow C$ ialah titik maksimum dan A ialah titik minimum.
C is a maximum point and A is a minimum point.



(Terima semua graf yang berbentuk serupa seperti di atas tanpa mengambil kira kedudukan graf.)
(Accept all graphs that have the similar shape regardless its position.)

3 (a) $\underset{x \to 3}{\text{had}} \frac{x-3}{4-\sqrt{19-x}} / \lim_{x \to 3} \frac{x-3}{4-\sqrt{19-x}}$

$= \underset{x \to 3}{\text{had}} \frac{(x-3)(4+\sqrt{19-x}}{16-(19-x)} / \lim_{x \to 3} \frac{(x-3)(4+\sqrt{19-x}}{16-(19-x)}$

$= \underset{x \to 3}{\text{had}} \frac{(x-3)(4+\sqrt{19-x}}{x-3} / \lim_{x \to 3} \frac{(x-3)(4+\sqrt{19-x}}{x-3}$

$= \underset{x \to 3}{\text{had}}\, 4 + \sqrt{19-x} / \lim_{x \to 3} 4 + \sqrt{19-x}$

$= 4 + \sqrt{19-3}$

$= 8$

(b) $y = 5x^0$

$\frac{dy}{dx} = 0(5x^{-1})$

$\therefore k = 0, m = -1$

4 $\frac{dy}{dx} = 4x + 7$

$4x + 7 = 5$

$x = -\frac{1}{2}$

$y = 2\left(-\frac{1}{2}\right)^2 + 7\left(-\frac{1}{2}\right) - 1$

$= -4$

$\therefore$ Koordinat/*Coordinates* $= \left(-\frac{1}{2}, -4\right)$

$y = 5x + p$ merupakan tangen kepada lengkung pada titik $\left(-\frac{1}{2}, -4\right)$ kerana $m_t = 5$.

$y = 5x + p$ is the tangent to the curve at point $\left(-\frac{1}{2}, -4\right)$ because $m_t = 5$.

$\therefore \quad y - (-4) = 5\left[x - \left(-\frac{1}{2}\right)\right]$

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$$y = 5x + \frac{5}{2} - 4$$
$$= 5x - \frac{3}{2}$$
$$\therefore \quad p = -\frac{3}{2}$$

Kertas 2

1 $\frac{dy}{dx} = 6x^2 + 1$

$x = 2, \frac{dy}{dx} = 6(2)^2 + 1$
$= 25$

$\delta x = 0.02p$

$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$

$\delta y \approx \frac{dy}{dx} \times \delta x$
$= 25 \times 0.02p$
$\delta y = 0.5p$

$\%\delta y = \frac{\delta y}{y} \times 100\%$
$= \frac{0.5p}{2(2^3) + 2} \times 100\%$
$= \frac{25}{9}p\%$

2
$$y_1 = y_2$$
$$x^2 - x - 5 = x^2 - \frac{31}{5}x + \frac{53}{5}$$
$$5x^2 - 5x - 25 = 5x^2 - 31x + 53$$
$$26x = 78$$
$$x = 3$$
$$y = (3)^2 - (3) - 5$$
$$= 1$$
$\therefore \quad A(3, 1)$

(a) $m_{t_1} = \frac{dy_1}{dx}$
$= 2x - 1$

$m_{t_2} = \frac{dy_2}{dx}$
$= 2x - \frac{31}{5}$

(b) Pada/*At* $x = 3$,
$m_{t_1} = 2(3) - 1$
$= 5$
$m_{t_2} = 2(3) - \frac{31}{5}$
$= -\frac{1}{5}$
$m_{t_1} \times m_{t_2} = 5 \times -\frac{1}{5}$
$= -1$

(Tertunjuk bahawa tangen kedua-dua lengkung itu ialah normal antara satu sama lain.)
(*Shown that the tangents of both curves are normal to each other.*)

3 $m_t = \frac{dy}{dx}$
$= 2x + 2$
$x = 2, m_t = 2(2) + 2$
$= 6$
$\therefore \; m_n = -\frac{1}{6}$

Persamaan normal/*Equation of normal:*
$y - 3 = -\frac{1}{6}(x - 2)$
$y = -\frac{1}{6}x + \frac{1}{3} + 3$
$y = -\frac{1}{6}x + \frac{10}{3}$
$\therefore \quad a = -\frac{1}{6}, b = \frac{10}{3}$

4 Biar/*Let* $y = x^{-\frac{1}{2}}$
$\frac{dy}{dx} = -\frac{1}{2}x^{-\frac{3}{2}}$
$x = 9, y = (9)^{-\frac{1}{2}}$
$= \frac{1}{3}$
$\frac{dy}{dx} = -\frac{1}{2}(9)^{-\frac{3}{2}}$
$= -\frac{1}{54}$

$\delta x = 0.02, \; \frac{\delta y}{\delta x} \approx \frac{dy}{dx}$
$\delta y \approx \frac{dy}{dx} \times \delta x$
$= -\frac{1}{54} \times 0.02$
$\delta y = -\frac{1}{2\,700}$
$\therefore \quad y_n = y_o + \delta y$
$(9.02)^{-\frac{1}{2}} = \frac{1}{3} - \frac{1}{2\,700}$
$= \frac{899}{2\,700}$

5 $PQ : PS : SR$
$= 2 : 5 : 3$
$= x : \frac{5x}{2} : \frac{3x}{2}$

(a) $A = \frac{1}{2}\left(x + \frac{3x}{2}\right)\left(\frac{5x}{2}\right)$
$= \frac{25x^2}{8}$

(b) (i) $\frac{dx}{dt} = 0.02 \text{ cm s}^{-1}$
$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$
$= \frac{25x}{4} \times 0.02$
$= \frac{25(4)}{4} \times 0.02$
$= 0.5 \text{ cm}^2 \text{ s}^{-1}$

(ii) $\delta x = 0.05$,
$\frac{\delta V}{\delta x} \approx \frac{dV}{dx}$
$\delta V \approx \frac{dV}{dx} \times \delta x$
$= \frac{15x^2}{4} \times 0.05$
$= \frac{15(4)^2}{4} \times 0.05$
$= 3 \text{ cm}^3$

$V = \frac{25x^2}{8} \times \frac{2}{5}x$
$= \frac{5x^3}{4}$
$\frac{dV}{dx} = \frac{15x^2}{4}$

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6 (a) $\theta = \frac{s}{r}$

Dalam sebutan r/*In terms of r*,

$\theta = \frac{26 - 2r}{r}$

(b) $\frac{dr}{dt} = 0.1$ cm s^{-1}, $r = 2$ cm,

(i) $\frac{d\theta}{dt} = \frac{d\theta}{dr} \times \frac{dr}{dt}$

$= \frac{r(-2) - (26 - 2r)(1)}{r^2} \times 0.1$

$= \frac{-2r - 26 + 2r}{r^2} \times 0.1$

$= \frac{-26}{(2)^2} \times 0.1$

$= -0.65$ rad s^{-1}

(ii) $A = \frac{1}{2}r^2\left(\frac{26 - 2r}{r}\right)$

$= \frac{26r - 2r^2}{2}$

$= 13r - r^2$

$\frac{dA}{dr} = 13 - 2r$

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$

$= [13 - 2(2)] \times 0.1$

$= 0.9$ cm^2 s^{-1}

7 (a) $V = 2\,666\frac{2}{3}$

$2x^2h = \frac{8\,000}{3}$

$h = \frac{4\,000}{3x^2}$

$A = 2(2x^2 + xh + 2xh)$

$= 4x^2 + 6xh$

$= 4x^2 + 6x\left(\frac{4\,000}{3x^2}\right)$

$= 4x^2 + \left(\frac{8\,000}{x}\right)$

A minimum apabila/*is minimum when*

$\frac{dA}{dx} = 0$

$8x - \frac{8\,000}{x^2} = 0$

$8x^3 - 8\,000 = 0$

$x^3 = 1\,000$

$x = 10$

$h = \frac{4\,000}{3(10)^2}$

$= 13\frac{1}{3}$

Dimensi kotak (panjang × lebar × tinggi)

Dimension of box (length × width × height)

$= 20$ cm $\times$ 10 cm $\times$ $13\frac{1}{3}$ cm

(b) Kos pengeluaran/*Production cost*

$= 20\,000 \times \left[4(10)^2 + \left(\frac{8\,000}{10}\right)\right] \times$ RM0.002

= RM48 000

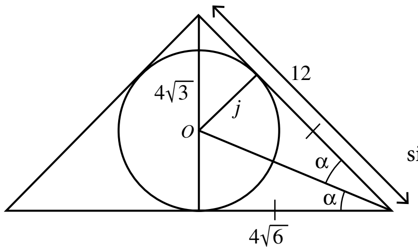
8 $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(12^2 - h^2)h$

$= \frac{1}{3}\pi(144 - h^2)h$

$= \frac{\pi}{3}(144h - h^3)$

V maksimum apabila/*V is maximum when* $\frac{dV}{dh} = 0$,



$\frac{\pi}{3}(144 - 3h^2) = 0$

$3h^2 = 144$

$h = \sqrt{48}$

$= 4\sqrt{3}$ cm

$r^2 = 144 - h^2$

$= 144 - 48$

$r = 4\sqrt{6}$ cm

$\sin 2\alpha = \frac{4\sqrt{3}}{12}$

$2\alpha = \sin^{-1}\frac{\sqrt{3}}{3}$

$= 35.26°$

$\alpha = 17.63°$

$\tan 17.63° = \frac{j}{4\sqrt{6}}$

$j = 3.114$ cm

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