Jawapan

Praktis 3

Praktis Formatif

1 (a) $\int 3x^2 + 24x - 3\,dx = x^3 + 12x^2 - 3x$
(b) $\int 12x^3 + 2x\,dx = 3x^4 + x^2 + 7$

2 (a) $\int 2 - 5x + 3x^2\,dx = \frac{2}{2}\int 2 - 5x + 3x^2\,dx$
$= \frac{1}{2}\int 4 - 10x + 6x^2\,dx$
$= \frac{1}{2}(4x - 5x^2 + 2x^3)$
$= 2x - \frac{5}{2}x^2 + x^3$

(b) $\int 6 + \frac{36}{x^5}dx = 3\int 2 + \frac{12}{x^5}dx$
$= 3\left(2x - \frac{3}{x^4}\right)$
$= 6x - \frac{9}{x^4}$

(c) $\int 2f(x)\,dx = 2\int f(x)\,dx$
$= 2g(x)$

(d) $\frac{d}{dx}\left(\frac{2x^2}{3x-1}\right) = \frac{(3x-1)\frac{d}{dx}(2x^2) - (2x^2)\frac{d}{dx}(3x-1)}{(3x-1)^2}$
$= \frac{(3x-1)(4x) - (2x^2)(3)}{(3x-1)^2}$
$= \frac{12x^2 - 4x - 6x^2}{(3x-1)^2}$
$= \frac{6x^2 - 4x}{(3x-1)^2}$
$= \frac{2x(3x-2)}{(3x-1)^2}$

$\int \frac{6x(3x-2)}{(3x-1)^2}dx = 3\int \frac{2x(3x-2)}{(3x-1)^2}dx$
$= 3\left(\frac{2x^2}{3x-1}\right)$
$= \frac{6x^2}{3x-1}$

3 (a) $C'(x) = 100t + 155$
(b) $\int 300t + 465\,dt = \int 3\,(100t + 155)\,dt$
$= 3\int (100t + 155)\,dt$
$= 3(50t^2 + 155t + 100)$
$C(28) = 3[50(28)^2 + 155(28) + 100]$
$=$ RM130 920

4 $f'(x)\int f'(x)dx = (4x^3 - 6x^2)(x^4 - 2x^3)$
$= 2x^2(2x-3)(x^3)(x-2)$
$= 2x^5(2x-3)(x-2)$

5 $y = \frac{2x-6}{x}$
$= 2 - 6x^{-1}$

(a) $\frac{dy}{dx} = 6x^{-2}$

(b) $4 + \int\left(\frac{dy}{dx}\right)dx = 0$
$4 + 2 - 6x^{-1} = 0$
$6 = 6x^{-1}$
$x = 1$

6 $\frac{3f(x)}{\int g(x)dx} = \frac{3f(x)}{f(x)}$
$= 3$

7 (a) $P_B'(5) = 2[2.72^{1.2(5)} - 2(5)]$
$= 789.9$
≈ 790

(b) $P_B(5) = 2\left[\frac{5}{6}(2.72^{1.2(5)}) - (5)^2 + 1\,495\right]$
$= 3\,614.9$
$\approx 3\,615$ orang/*people*

8 (a) $\int 3\,dx = 3x + c$
(b) $\int 24x\,dx = 12x^2 + c$
(c) $\int 6x^2\,dx = 2x^3 + c$
(d) $\int 3x^2 + 4x\,dx = x^3 + 2x^2 + c$
(e) $\int \frac{2}{x^4}dx = \int 2x^{-4}\,dx$
$= \frac{2x^{-4+1}}{-3} + c$
$= -\frac{2}{3x^3} + c$
(f) $\int x^2(x-3)\,dx = \int x^3 - 3x^2\,dx$
$= \frac{x^4}{4} - x^3 + c$
(g) $\int (x+2)(2x^4 - 1)\,dx = \int 2x^5 - x + 4x^4 - 2\,dx$
$= \frac{2x^6}{6} - \frac{x^2}{2} + \frac{4x^5}{5} - 2x + c$
$= \frac{x^6}{3} + \frac{4x^5}{5} - \frac{x^2}{2} - 2x + c$
(h) $\int \frac{x^2 + 3x + 2}{x+2}\,dx = \int \frac{(x+2)(x+1)}{x+2}\,dx$
$= \int x + 1\,dx$
$= \frac{x^2}{2} + x + c$

9 (a) (i) Biar/*Let* $u = x + 2$, $\frac{du}{dx} = 1$
$\int \frac{2}{(x+2)^5}\,dx = \int 2u^{-5}\,du$
$= \frac{2u^{-4}}{-4} + c$
$= -\frac{1}{2(x+2)^4} + c$

(ii) Biar/*Let* $u = 3x + 2$, $\frac{du}{dx} = 3$
$\int \frac{1}{3}(3x+2)^8\,dx = \frac{3}{5}\int \frac{u^8}{3}\,du$

$$= \frac{u^9}{5(9)} + c$$
$$= -\frac{(3x+2)^9}{45} + c$$

(b) (i) $\int \frac{2}{(x+2)^5} dx = \int 2(x+2)^{-5}\, du$
$$= \frac{2(x+2)^{-4}}{(-4)(1)} + c$$
$$= -\frac{1}{2(x+2)^4} + c$$

(ii) $\int \frac{3}{5}(3x+2)^8\, dx = \frac{3}{5}\left[\frac{(3x+2)^9}{9(3)}\right] + c$
$$= \frac{(3x+2)^9}{45} + c$$

10 (a) $\frac{dy}{dx} = 3x^2 + x - 2$

$y = \int 3x^2 + x - 2\, dx$

$= x^3 + \frac{x^2}{2} - 2x + c$

Pada/*At* $P(2, 15)$, $15 = (2)^3 + \frac{(2)^2}{2} - 2(2) + c$

$c = 9$

$y = x^3 + \frac{x^2}{2} - 2x + 9$

(b) $f(x) = \int 2x + 9\, dx$

$= x^2 + 9x + c$

$f(3) = 21$

$(3)^2 + 9(3) + c = 21$

$c = -15$

$f(x) = x^2 + 9x - 15$

(c) $g(t) = \frac{(t-1)(5t-1)}{t^3(t-1)}$

$= 5t^{-2} - t^{-3}$

$y = \int 5t^{-2} - t^{-3}\, dt$

$= \frac{5t^{-1}}{(-1)} - \frac{t^{-2}}{(-2)} + c$

$= -\frac{5}{t} + \frac{1}{2t^2} + c$

$3 = -\frac{5}{(1)} + \frac{1}{2(1)^2} + c$

$c = \frac{15}{2}$

$y = \frac{15}{2} - \frac{5}{t} + \frac{1}{2t^2}$

11 $\frac{ds}{dt} = t^2 + 9$

$s = \int t^2 + 9\, dt$

$= \frac{t^3}{3} + 9t + c$

$4 = \frac{(3)^3}{3} + 9(3) + c$

$c = -32$

$s = \frac{t^3}{3} + 9t - 32$

$s = \frac{(10)^3}{3} + 9(10) - 32$

$= 391\frac{1}{3}$ m

12 $y = \int kx^2 + 2x\, dx$

$= \frac{kx^3}{3} + x^2 + c$

Pada/*At* $A(1, 6)$,

$6 = \frac{k}{3} + 1 + c$

$15 = k + 3c$

$k + 3c = 15 \ldots$ ①

Pada/*At* $B(-2, 0)$,

$0 = \frac{k(-2)^3}{3} + (-2)^2 + c$

$0 = -8k + 12 + 3c$

$8k - 3c = 12 \ldots$ ②

① + ②: $9k = 27$

$k = 3$

$3c = 15 - 3$

$c = 4$

$y = \frac{3x^3}{3} + x^2 + 4$

$y = x^3 + x^2 + 4$

13 (a) $\int_1^2 \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$

$= \int_1^2 (x^{\frac{1}{2}} + x^{-\frac{1}{2}})\, dx$

$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]$

$= \left[\frac{2x^{\frac{3}{2}}}{3} + 2x^{\frac{1}{2}}\right]_1^2$

$= \left[\frac{2(2)^{\frac{3}{2}}}{3} + 2(2)^{\frac{1}{2}}\right] - \left[\frac{2(1)^{\frac{3}{2}}}{3} + 2(1)^{\frac{1}{2}}\right]$

$= 2.0474$

(b) $\int_0^3 \left(\frac{x^4 + 3x}{x}\right) dx = \int_0^3 x^3 + 3\, dx$

$= \left[\frac{x^4}{4} + 3x\right]_0^3$

$= \frac{(3)^4}{4} + 3(3) - 0$

$= 29.25$

(c) $\int_{-2}^{-1} \left(\frac{(4-x)(3-x)}{x^5}\right) dx$

$= \int_{-2}^{-1} \left(\frac{12 - 7x + x^2}{x^5}\right) dx$

$= \int_{-2}^{-1} (12x^{-5} - 7x^{-4} + x^{-3})\, dx$

$= \left[-3x^{-4} + \frac{7x^{-3}}{3} - \frac{x^{-2}}{2}\right]_{-2}^{-1}$

$= \left[-\frac{3}{x^4} + \frac{7}{3x^3} - \frac{1}{2x^2}\right]_{-2}^{-1}$

$= \left(-\frac{3}{(-1)^4} + \frac{7}{3(-1)^3} - \frac{1}{2(-1)^2}\right)$
$- \left(-\frac{3}{(-2)^4} + \frac{7}{3(-2)^3} - \frac{1}{2(-2)^2}\right)$

$= -5\frac{11}{48}$

14 (a) $\int_a^b 3f(x)\, dx = 3\int_a^b f(x)\, dx$

$= 3(5)$

$= 15$

(b) $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$
$= 5 + 8$
$= 13$

(c) $\int_a^b [f(x) - g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$
$= 5 - 2$
$= 3$

(d) $\int_c^a f(x)\,dx = -\int_a^c f(x)\,dx$
$= -13$

(e) $\int_a^b [g(x) + 3]\,dx = \int_a^b g(x)\,dx + \int_a^b 3\,dx$
$= 2 + 3b - 3a$

(f) $\int_a^a f(x)\,dx = 0$

(g)
$$\int_b^a [f(x) + kx]\,dx = 25$$
$$\int_b^a f(x)\,dx + \int_b^a kx\,dx = 25$$
$$-\int_a^b f(x)\,dx + k\left[\frac{x^2}{2}\right]_4^1 = 25$$
$$-5 + k\left(\frac{(1)^2 - (4)^2}{2}\right) = 25$$
$$\frac{15}{2}k = 30$$
$$k = -4$$

15 (a) $A = \int_2^4 x^3 - 4x^2 + x + 10\,dx$
$= \left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{x^2}{2} + 10x\right]_2^4$
$= \left[\frac{(4)^4}{4} - \frac{4(4)^3}{3} + \frac{(4)^2}{2} + 10(4)\right] - \left[\frac{(2)^4}{4} - \frac{4(2)^3}{3} + \frac{(2)^2}{2} + 10(2)\right]$
$= 11\frac{1}{3}$ unit²/*units*²

(b) $A = \int_{-2}^5 (3x + 10)^{\frac{1}{2}}\,dx$
$= \left[\frac{(3x + 10)^{\frac{3}{2}}}{\frac{3}{2}(3)}\right]_{-2}^5$
$= \frac{2}{9}\left[(3x + 10)^{\frac{3}{2}}\right]_{-2}^5$
$= \frac{2}{9}\left\{[3(5) + 10]^{\frac{3}{2}} - [3(-2) + 10]^{\frac{3}{2}}\right\}$
$= 26$ unit²/*units*²

(c) $A = \left|\int_3^6 x^2 - 9x + 18\,dx\right|$
$= \left|\left[\frac{x^3}{3} - \frac{9x^2}{2} + 18x\right]_3^6\right|$
$= \left|\left[\frac{(6)^3}{3} - \frac{9(6)^2}{2} + 18(6)\right] - \left[\frac{(3)^3}{3} - \frac{9(3)^2}{2} + 18(3)\right]\right|$
$= |-4.5|$
$= 4.5$ unit²/*units*²

(d)
$$y = x^2 - 4x$$
$$5 = x^2 - 4x$$
$$x^2 - 4x - 5 = 0$$
$$(x - 5)(x + 1) = 0$$
$$x = -1, 5$$

$A = \int_{-1}^0 x^2 - 4x\,dx + \left|\int_0^3 x^2 - 4x\,dx\right| + \int_4^5 x^2 - 4x\,dx$
$= \left[\frac{x^3}{3} - 2x^2\right]_{-1}^0 + \left|\left[\frac{x^3}{3} - 2x^2\right]_0^3\right| + \left[\frac{x^3}{3} - 2x^2\right]_4^5$
$= 0 - \left[\frac{(-1)^3}{3} - 2(-1)^2\right] + \left|\left[\frac{(3)^3}{3} - 2(3)^2\right] - 0\right| + \left[\frac{(5)^3}{3} - 2(5)^2\right] - \left[\frac{(4)^3}{3} - 2(4)^2\right]$
$= 0 - \left(-2\frac{1}{3}\right) + |-9| + \left(-8\frac{1}{3}\right) - \left(-10\frac{2}{3}\right)$
$= 13\frac{2}{3}$ unit²/*units*²

16 (a) $A = \int_1^5 \frac{1}{2}(5 - y)\,dy$
$= \frac{1}{2}\left[5y - \frac{y^2}{2}\right]_1^5$
$= \frac{1}{2}\left\{\left[5(5) - \frac{5^2}{2}\right] - \left[5(1) - \frac{1^2}{2}\right]\right\}$
$= 4$ unit²/*units*²

(b) $A = \int_{-2}^0 (y + 2)^2 - 4\,dy$
$= \left|\left[\frac{(y + 2)^3}{3} - 4y\right]_{-2}^0\right|$
$= \left|\left[\frac{(0 + 2)^3}{3} - 0\right] - \left[\frac{(-2 + 2)^3}{3} - 4(-2)\right]\right|$
$= \left|-5\frac{1}{3}\right|$
$= 5\frac{1}{3}$ unit²/*units*²

(c) $A = \int_{-1}^1 y^2 + 3\,dy$
$= \left[\frac{(y)^3}{3} + 3y\right]_{-1}^1$
$= \left[\frac{(1)^3}{3} + 3(1)\right] - \left[\frac{(-1)^3}{3} + 3(-1)\right]$
$= 6\frac{2}{3}$ unit²/*units*²

(d) $A = \int_{-1}^0 2y^3 - y^2 - 6y\,dy + \left|\int_0^1 2y^3 - y^2 - 6y\,dy\right|$
$= \left[\frac{2y^4}{4} - \frac{y^3}{3} - 3y^2\right]_{-1}^0 + \left|\left[\frac{2y^4}{4} - \frac{y^3}{3} - 3y^2\right]_0^1\right|$
$= 0 - \left[\frac{(-1)^4}{2} - \frac{(-1)^3}{3} - 3(-1)^2\right] + \left|\left[\frac{(1)^4}{2} - \frac{(1)^3}{3} - 3(1)^2\right] - 0\right|$
$= 2\frac{1}{6} + \left|-2\frac{5}{6}\right|$
$= 5$ unit²/*units*²

17 (a)
$$x = 4x - x^2$$
$$x^2 + x - 4x = 0$$
$$x^2 - 3x = 0$$
$$x = 0, 3$$

$A = \int_0^3 4x - x^2 - x\,dy$
$= \int_0^3 3x - x^2\,dy$

$= \left[\frac{3}{2}x^2 - \frac{1}{3}x^3\right]_0^3$

$= \frac{3}{2}(3)^2 - \frac{1}{3}(3)^3 - 0$

$= 4.5$ unit²/*units*²

(b) $x^2 - x + 1 = 2x + 1$

$x^2 - 3x = 0$

$x = 0, 3$

$A = \int_0^3 2x + 1 - (x^2 - x + 1)\, dy$

$= \int_0^3 2x + 1 - x^2 + x - 1\, dy$

$= \int_0^3 3x - x^2\, dy$

$= \left[\frac{3}{2}x^2 - \frac{1}{3}x^3\right]_0^3$

$= \frac{3}{2}(3)^2 - \frac{1}{3}(3)^3 - 0$

$= 4.5$ unit²/*units*²

(c) $\frac{x}{2} = x^2 - 6x + 9$

$x = 2x^2 - 12x + 18$

$2x^2 - 13x + 18 = 0$

$(2x - 9)(x - 2) = 0$

$x = 2, 4.5$

$x^2 - 6x + 9 = 0$

$(x - 3)^2 = 0$

$x = 3$

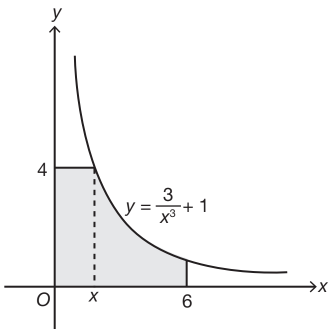
$A = \int_0^2 \frac{x}{2} dx + \int_2^3 x^2 - 6x + 9\, dx$

$= \left[\frac{x^2}{4}\right]_0^2 + \left[\frac{1}{3}x^3 - 3x^2 + 9x\right]_2^3$

$= \left[\frac{2^2}{4} - 0\right] + \left[\frac{1}{3}(3)^3 - 3(3)^2 + 9(3)\right] - \left[\frac{1}{3}(2)^3 - 3(2)^2 + 9(2)\right]$

$= 1\frac{1}{3}$ unit²/*units*²

(d)



$4 = \frac{3}{x^3} + 1$

$3x^3 = 3$

$x^3 = 1$

$x = 1$

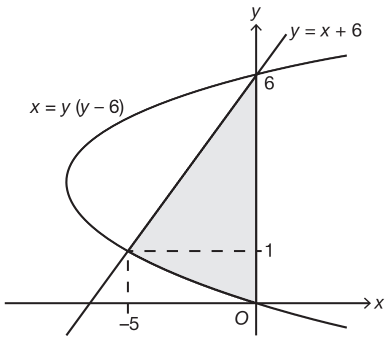
$A = (1)(4) + \int_1^6 3x^{-3} + 1\, dx$

$= 4 + \left[\frac{3x^{-3+1}}{(-2)} + x\right]_1^6$

$= 4 + \left[-\frac{3}{2(6)^2} + (6)\right] - \left[-\frac{3}{2(1)^2} + (1)\right]$

$= 10\frac{11}{24}$ unit²/*units*²

18



$y^2 - 6y = y - 6$

$y^2 - 7y + 6 = 0$

$(y - 1)(y - 6) = 0$

$y = 1, 6$

$x = 1 - 6$

$= -5$

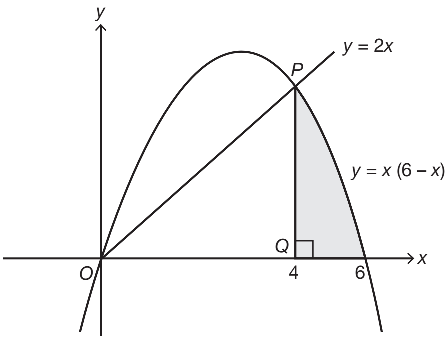
$A = \left|\int_0^1 y^2 - 6y\, dy\right| + \frac{1}{2}(5)(5)$

$= \left|\left[\frac{y^3}{3} - 3y^2\right]_0^1\right| + \frac{25}{2}$

$= \left|\left[\frac{1^3}{3} - 3(1)^2\right] - 0\right| + \frac{25}{2}$

$= 15\frac{1}{6}$ unit²/*units*²

19



$6x - x^2 = 2x$

$x^2 - 4x = 0$

$x(x - 4) = 0$

$x = 0, 4$

$A = \int_4^6 6x - x^2\, dx$

$= \left[3x^2 - \frac{x^3}{3}\right]_4^6$

$= \left[3(6)^2 - \frac{(6)^3}{3}\right] - \left[3(4)^2 - \frac{(4)^3}{3}\right]$

$= 9\frac{1}{3}$ unit²/*units*²

20 (a) $V = \pi\int_0^3 (x^2+2)^2\,dx$

$= \pi\int_0^3 x^4 + 4x^2 + 4\,dx$

$= \pi\left[\frac{x^5}{5} + \frac{4x^3}{3} + 4x\right]_0^3$

$= \pi\left(\frac{3^5}{5} + \frac{4(3)^3}{3} + 4(3) - 0\right)$

$= 96.6\pi$ unit^3/*units*^{3}

(b) $V = \pi\int_4^9 (4+\sqrt{x})^2\,dx$

$= \pi\int_4^9 16 + 8\sqrt{x} + x\,dx$

$= \pi\left[16x + \frac{8x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^2}{2}\right]_4^9$

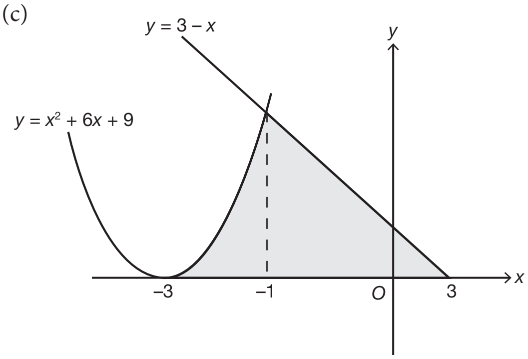
$= \pi\left[16x + \frac{16x^{\frac{3}{2}}}{3} + \frac{x^2}{2}\right]_4^9$

$= \pi\left\{\left[16(9) + \frac{16(9)^{\frac{3}{2}}}{3} + \frac{(9)^2}{2}\right] - \left[16(4) + \frac{16(4)^{\frac{3}{2}}}{3} + \frac{(4)^2}{2}\right]\right\}$

$= 213\frac{5}{6}\pi$ unit^3/*units*^{3}

$= 213.83\pi$ unit^3/*units*^{3}

(c)



$x^2 + 6x + 9 = 0$

$(x+3)^2 = 0$

$x = -3$

$x^2 + 6x + 9 = 3 - x$

$x^2 + 7x + 6 = 0$

$(x+6)(x+1) = 0$

$x = -1, -6$

$3 - x = 0$

$x = 3$

$V = \pi\int_{-3}^{-1} (x^2+6x+9)^2\,dx + \int_{-1}^{3} \pi(3-x)^2\,dx$

$= \pi\int_{-3}^{-1} x^4 + 12x^3 + 54x^2 + 108x + 81\,dx + \pi\int_{-1}^{3} 9 - 6x + x^2\,dx$

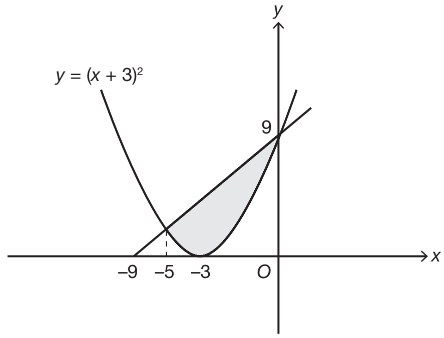
$= \pi\left[\frac{x^5}{5} + 3x^4 + 18x^3 + 54x^2 + 81x\right]_{-3}^{-1} + \pi\left[9x - 3x^2 + \frac{x^3}{3}\right]_{-1}^{3}$

$= \pi\left[\frac{(1)^5}{5} + 3(-1)^4 + 18(-1)^3 + 54(-1)^2 + 81(-1)\right] - \pi\left[\frac{(-3)^5}{5} + 3(-3)^4 + 18(-3)^3 + 54(-3)^2 + 81(-3)\right] + \pi\left[9(3) - 3(3)^2 + \frac{(3)^3}{3}\right] - \pi\left[9(-1) - 3(-1)^2 + \frac{(-1)^3}{3}\right]$

$= 6\frac{2}{5}\pi + 21\frac{1}{3}\pi$

$= 27\frac{11}{15}\pi$ unit^3/*units*^{3}

(d)



$y = x + 9$

$x^2 + 6x + 9 = x + 9$

$x^2 + 5x = 0$

$x(x+5) = 0$

$x = -5, 0$

$V = \pi\int_{-5}^{0} (x+9)^2\,dx - \pi\int_{-5}^{0} (x+3)^4\,dx$

$= \pi\int_{-5}^{0} x^2 + 18x + 81\,dx - \pi\int_{-5}^{0} x^4 + 12x^3 + 54x^2 + 108x + 81\,dx$

$= \pi\left[\frac{x^3}{3} + 9x^2 + 81x\right]_{-5}^{0} - \pi\left[\frac{x^5}{5} + 3x^4 + 18x^3 + 54x^2 + 81x\right]_{-5}^{0}$

$= 0 - \pi\left[\frac{(-5)^3}{3} + 9(-5)^2 + 81(-5)\right] - \pi\left\{0 - \left[\frac{(-5)^5}{5} + 3(-5)^4 + 18(-5)^3 + 54(-5)^2 + 81(-5)\right]\right\}$

$= 221\frac{2}{3}\pi - (55)\pi$

$= 166\frac{2}{3}\pi$ unit^3/*units*^{3}

21 (a) $V = \pi\int_2^{10} y - 1\,dy$

$= \pi\left[\frac{y^2}{2} - y\right]_2^{10}$

$= \pi\left[\left(\frac{10^2}{2} - 10\right) - \left(\frac{2^2}{2} - 2\right)\right]$

$= 40\pi$ unit^3/*units*^{3}

(b) $y = mx + c$

$4 = 2m + 10$

$m = -3$

$y = 10 - 3x$

$x = \frac{1}{3}(10 - y)$

$x^2 = \frac{1}{9}(10 - y)^2$

$V = \pi\int_0^4 y\, dy + \pi\int_4^{10} \frac{1}{9}(10 - y)^2\, dy$

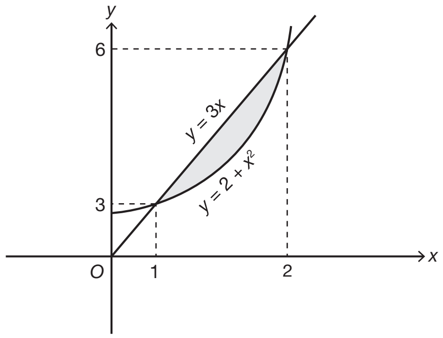
$= \pi\left[\frac{y^2}{2}\right]_0^4 + \frac{1}{9}\pi\left[\frac{(10 - y)^3}{3(-1)}\right]_4^{10}$

$= \pi\left(\frac{4^2}{2} - 0\right) - \frac{1}{27}\pi[(10 - 10)^3 - (10 - 4)^3]$

$= 8\pi + 8\pi$

$= 16\pi$ unit^3/*units*^{3}

(c)



$x = 1, y = 3$

$x = 2, y = 6$

$3x = y$

$x^2 = \frac{y^2}{9}$

$x^2 = y - 2$

$V = \pi\int_3^6 y - 2 - \frac{y^2}{9}\, dy$

$= \pi\left[\frac{y^2}{2} - 2y - \frac{y^3}{27}\right]_3^6$

$= \pi\left[\frac{(6)^2}{3} - 2(6) - \frac{(6)^3}{27}\right] - \pi\left[\frac{(3)^2}{3} - 2(3) - \frac{(3)^3}{27}\right]$

$= \frac{1}{2}\pi$ unit^3/*units*^{3}

22 $V = \pi\int_2^k 64x^{-2}\, dx$

$= 64\pi\left[\frac{x^{-1}}{(-1)}\right]_2^k$

$= 64\pi\left[-\frac{1}{x}\right]_2^k$

$= 64\pi\left[-\frac{1}{k} - \left(-\frac{1}{2}\right)\right]$

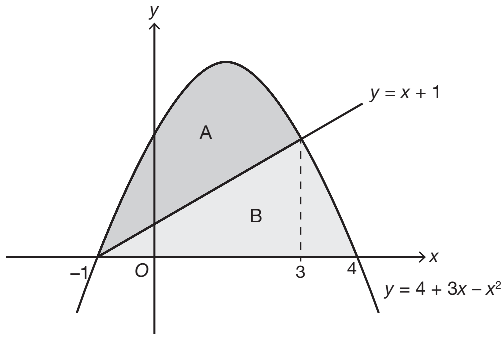
$= 64\pi\left(\frac{1}{2} - \frac{1}{k}\right)$

$= 32\pi - \frac{64\pi}{k}$

$k \to \infty, \frac{1}{k} \approx 0$

$\therefore V \approx 32\pi$ unit^3/*units*^{3}

23



$-(x^2 - 3x - 4) = 0$

$(x - 4)(x + 1) = 0$

$x = -1, 4$

$x + 1 = 4 + 3x - x^2$

$x^2 - 2x - 3 = 0$

$(x - 3)(x + 1) = 0$

$x = -1, 3$

$A_A = \int_{-1}^3 4 + 3x - x^2 - (x + 1)\, dx$

$= \int_{-1}^3 3 + 2x - x^2\, dx$

$= \left[3x + x^2 - \frac{1}{3}x^3\right]_{-1}^3$

$= \left[3(3) + (3)^2 - \frac{1}{3}(3)^3] - [3(-1) + (-1)^2 - \frac{1}{3}(-1)^3\right]$

$= 10\frac{2}{3}$ unit^2/*units*^{2}

$A_B = \int_{-1}^3 x + 1\, dx + \int_3^4 4 + 3x - x^2\, dx$

$= \left[\frac{x^2}{2} + x\right]_{-1}^3 + \left[4x + \frac{3x^2}{2} - \frac{x^3}{3}\right]_3^4$

$= \left[\frac{(3)^2}{2} + (3)\right] - \left[\frac{(-1)^2}{2} + (-1)\right] + \left[4(4) + \frac{3(4)^2}{2} - \frac{(4)^3}{3}\right] - \left[4(3) + \frac{3(3)^2}{2} - \frac{(3)^3}{3}\right]$

$= 10\frac{1}{6}$ unit^2/*units*^{2}

$A : B = 10\frac{2}{3} : 10\frac{1}{6}$

$= 64 : 61$

24 $x^2 - 1 = 0$

$(x + 1)(x - 1) = 0$

$x = -1, 1$

$\therefore P(-1, 0), Q(1, 0)$

$A = \left|\int_{-1}^1 x^2 - 1\, dx\right| + \int_{-1}^1 3 + 2x - x^2\, dx$

$= \left|\left[\frac{x^3}{3} - x\right]_{-1}^1\right| + \left[3x + x^2 - \frac{x^3}{3}\right]_{-1}^1$

$= \left|\left[\frac{1^3}{3} - 1\right] - \left[\frac{(-1)^3}{3} - (-1)\right]\right| + \left[3(1) + (1)^2 - \frac{1^3}{3}\right] - \left[3(-1) + (-1)^2 - \frac{(-1)^3}{3}\right]$

$= 6\frac{2}{3}$ unit^2/*units*^{2}

25 $V = \frac{1}{2}\pi\int_1^4 (3x^2)^2\, dx$
$= \frac{9}{2}\pi\int_1^4 x^4\, dx$
$= \frac{9}{2}\pi\left[\frac{x^5}{5}\right]_1^4$
$= \frac{9}{2}\pi\left[\frac{4^5}{5} - \frac{1^5}{5}\right]$
$= 920\frac{7}{10}\pi$ unit^3/*units*^{3}

26 $\frac{dh}{dt} = 0.56t \text{ cms}^{-1}$
$h = \int 0.56t\, dt$
$= 0.28t^2 + c$
$t = 0.\ h = 0,\ \therefore\ c = 0$
$h = 0.28t^2$
$28 = 0.28t^2$
$t^2 = 100$
$t = 10$

27 $\frac{ds}{dt} = 3 - 9.8t$
$s = \int 3 - 9.8t\, dt$
$= 3t - 4.9t^2 + c$
$t = 0.\ s = 0,\ \therefore\ c = 0$
$s = 3t - 4.9t^2$

(a) Tinggi maksimum dicapai apabila
The maximum height is achieved when
$\frac{ds}{dt} = 0$
$3 - 9.8t = 0$
$t = \frac{15}{49}$ s
$s = 3\left(\frac{15}{49}\right) - 4.9\left(\frac{15}{49}\right)^2$
$= \frac{45}{98}$m

(b) $s = 0$
$3t - 4.9t^2 = 0$
$t(3 - 4.9t) = 0$
$t = 0, \frac{30}{49}$
$\therefore\ t = \frac{30}{49}$ s

28 (a) $(1 - y^2)^2 = 1 - 2y^2 + y^4$
$(0.8 - 0.5y^2)^2 = 0.64 - 0.8y^2 + 0.25y^4$
$V = \pi\int_{-0.2}^{0.2} (1 - 2y^2 + y^4) - (0.64 - 0.8y^2 + 0.25y^4)\, dy$
$= \pi\int_{-0.2}^{0.2} 0.36 - 1.2y^2 + 0.75y^4\, dy$
$= \pi[0.36y - 0.4y^3 + 0.15y^5]_{-0.2}^{0.2}$
$= \pi[0.36(0.2) - 0.4(0.2)^3 + 0.15(0.2)^5]$
$- \pi[0.36(-0.2) - 0.4(-0.2)^3 + 0.15(-0.2)^5]$
$= 0.1377\pi \text{ cm}^3$

(b) Jisim/*Mass* $= 4.51 \times 0.1377\pi$
$= 1.9513$ g
Harga/*Price* = RM153.49 × 1.9513
= RM299.51

Praktis Sumatif

Kertas 1

1 $\int_m^2 (2x + 3)\, dx = -8$
$[x^2 + 3x]_m^2 = -8$
$[2^2 + 3(2)] - [m^2 + 3m] = -8$
$10 - m^2 - 3m = -8$
$m^2 + 3m - 18 = 0$
$(m + 6)(m - 3) = 0$
$m = 3$ (−6 tidak diterima)
(−6 *not accepted*)

2 $y = \int 5x + 3\, dx$
$= \frac{5x^2}{2} + 3x + c$
$4 = 0 + 0 + c$
$c = 4$
$y = \frac{5x^2}{2} + 3x + 4$

3 $\int \frac{(2x + 3)^3}{2x^5}\, dx = \int \frac{8x^3 + 36x^2 + 54x + 27}{2x^5}\, dx$
$= \frac{1}{2}\int 8x^{-2} + 36x^{-3} + 54x^{-4} + 27x^{-5}\, dx$
$= \frac{1}{2}\left[\frac{8x^{-1}}{(-1)} + \frac{36x^{-2}}{(-2)} + \frac{54x^{-3}}{(-3)} + \frac{27x^{-4}}{(-4)} + c\right]$
$= \frac{1}{2}\left[-\frac{8}{x} - \frac{18}{x^2} - \frac{18}{x^3} - \frac{27}{4x^4} + c\right]$
$= -\frac{4}{x} - \frac{9}{x^2} - \frac{9}{x^3} - \frac{27}{8x^4} + c$

4 $\int \frac{3}{(3x - 2)^5}\, dx = 3\int (3x - 2)^{-5}\, dx$
$= 3\left[\frac{(3x - 2)^{-4}}{3(-4)}\right] + c$
$= -\frac{1}{4}(3x - 2)^{-4} + c$

Bandingkan dengan/*Compare with* $a(3x - 2)^n + c$,

(a) $a = -\frac{1}{4}, n = -4$

(b) $\int_0^2 \frac{3}{(3x - 2)^5}\, dx = \left[-\frac{1}{4(3x - 2)^4}\right]_0^2$
$= \left[-\frac{1}{4(6 - 2)^4}\right]_0^2 - \left[-\frac{1}{4(0 - 2)^4}\right]$
$= \frac{15}{1\,024}$

5 $\int_0^b f(x)\, dx = 20$
$\int_0^a f(x)\, dx + \int_a^b f(x)\, dx = 20$
$-x + 3x = 20$
$2x = 20$
$x = 10$
$\int_a^b f(x)\, dx = 3(10)$
$= 30$ unit^2/*units*^{2}

Kertas 2

1 $y = 2x^4\sqrt{4x - 3}$
$\frac{dy}{dx} = \sqrt{4x - 3}\frac{d}{dx}(2x^4) + (2x^4)\frac{d}{dx}\sqrt{4x - 3}$
$= \sqrt{4x - 3}(8x^3) + \frac{4x^4}{\sqrt{4x - 3}}$

$$= \frac{(4x - 3)(8x^3) + 4x^4}{\sqrt{4x - 3}}$$
$$= \frac{32x^4 - 24x^3 + 4x^4}{\sqrt{4x - 3}}$$
$$= \frac{36x^4 - 24x^3}{\sqrt{4x - 3}}$$
$$\int \frac{3x^4 - 2x^3}{\sqrt{4x - 3}}\, dx = \frac{1}{12}\int \frac{36x^4 - 24x^3}{\sqrt{4x - 3}}\, dx$$
$$= \frac{1}{12}(2x^4\sqrt{4x - 3})$$
$$= \frac{x^4}{6}\sqrt{4x - 3}$$

2 (a) $\frac{dV_1}{dt} = 6t^2 + 10t$

(b) $\frac{dV_2}{dt} = \frac{1}{2}t(3t + 5)$
$$= \frac{1}{4}\,\frac{dV_1}{dt}$$
$$V_2 = \int_5^6 \frac{1}{4}(6t^2 + 10t)\, dt$$
$$= \frac{1}{4}[2t^3 + 5t^2 + 9]_5^6$$
$$= \frac{1}{4}\{[2(6)^3 + 5(6)^2 + 9] - [2(5)^3 + 5(5)^2 + 9]\}$$
$$= 59\frac{1}{4}\text{ cm}^3$$

3 (a) $m_n = \frac{x^2}{x^2 + 1}$
$$m_t = -\frac{x^2 + 1}{x^2}$$
$$y = \int -\frac{x^2 + 1}{x^2}\, dx$$
$$= \int -1 - x^{-2}\, dx$$
$$= -x - \frac{x^{-1}}{(-1)} + c$$
$$= -x + \frac{1}{x} + c$$
$$5 = -1 + 1 + c$$
$$c = 5$$
$$\therefore y = -x + \frac{1}{x} + 5$$
$$= \frac{-x^2 + 1 + 5x}{x}$$
$$= \frac{1 + 5x - x^2}{x} \text{ (tertunjuk/\textit{shown})}$$

(b) Pada/*At* $x = 2$,
$$y = \frac{1 + 5(2) - (2)^2}{2}$$
$$= \frac{7}{2}$$
$$m_t = -\frac{x^2 + 1}{x^2}$$
$$= -\frac{2^2 + 1}{2^2}$$
$$= -\frac{5}{4}$$
$$y - \frac{7}{2} = -\frac{5}{4}(x - 2)$$
$$4y - 14 = -5x + 10$$
$$5x + 4y - 24 = 0$$

4 (a) Pada titik pegun/*At the stationary points,*
$$f'(x) = 0$$
$$3x^2 + mx + n = 0$$
$x = 1$, $\quad 3 + m + n = 0 \ldots ①$

$x = -3$, $\quad 27 - 3m + n = 0 \ldots ②$

① – ②: $\quad -24 + 4m = 0$
$$m = 6$$
$$n = -3 - 6$$
$$= -9$$

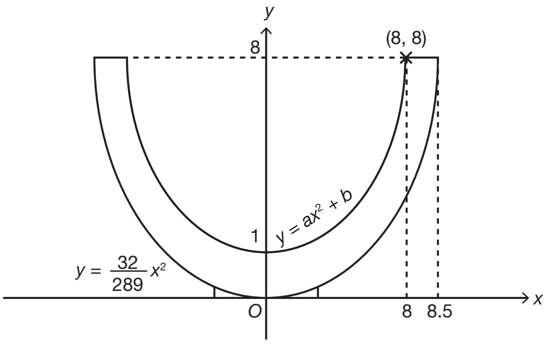
(b) $f'(x) = 3x^2 + 6x - 9$
$$f(x) = \int 3x^2 + 6x - 9\, dx$$
$$= x^3 + 3x^2 - 9x + c$$
$$-3 = (1)^3 + 3(1)^2 - 9(1) + c$$
$$c = 2$$
$$f(x) = x^3 + 3x^2 - 9x + 2$$

5 (a) $A = \int_0^a \left(\frac{x}{a}\right)^3 dx$
$$= \frac{1}{a^3}\int_0^a x^3\, dx$$
$$= \frac{1}{a^3}\left[\frac{x^4}{4}\right]_0^a$$
$$= \frac{1}{a^3}\left(\frac{a^4}{4} - 0\right)$$
$$= \frac{a}{4}$$

(b) $B = \int_a^b \left(\frac{a}{x}\right)^3 dx$
$$= a^3\int_0^a x^{-3}\, dx$$
$$= a^3\left[\frac{x^{-2}}{-2}\right]_a^b$$
$$= a^3\left\{\left[-\frac{1}{2b^2}\right] - \left[-\frac{1}{2a^2}\right]\right\}$$
$$= a^3\left(\frac{b^2 - a^2}{2a^2b^2}\right)$$
$$= \frac{ab^2 - a^3}{2b^2}$$

(c) $A = \frac{a}{4}, B = \frac{a}{2}\left(1 - \frac{a^2}{b^2}\right)$
$$B = 2\left[\frac{a}{4}\left(1 - \frac{a^2}{b^2}\right)\right]$$
$$= 2A\left(1 - \frac{a^2}{b^2}\right)$$
$$\frac{B}{2A} = 1 - \frac{a^2}{b^2}$$
$0 < a < b$, $\quad \therefore 1 - \frac{a^2}{b^2} < 1$
$$\frac{B}{2A} < 1$$
$$\frac{B}{2} < A$$
$$A > \frac{1}{2}B \text{ (tertunjuk/\textit{shown})}$$

6 (a) $y = ax^2 + b$
$$b = 1$$
$$8 = a(8)^2 + 1$$
$$64a = 7$$
$$a = \frac{7}{64}$$



(b) $y = \frac{7}{64}x^2 + 1$

$x^2 = \frac{64}{7}(y - 1)$

$V = \frac{64}{7}\pi\int_1^8 (y - 1)\, dy$

$= \frac{64}{7}\pi\left[\frac{y^2}{2} - y\right]_1^8$

$= \frac{64}{7}\pi\left\{\left[\frac{(8)^2}{2} - (8)\right] - \left[\frac{1^2}{2} - 1\right]\right\}$

$= \frac{64}{7}\pi\left(24\frac{1}{2}\right)$

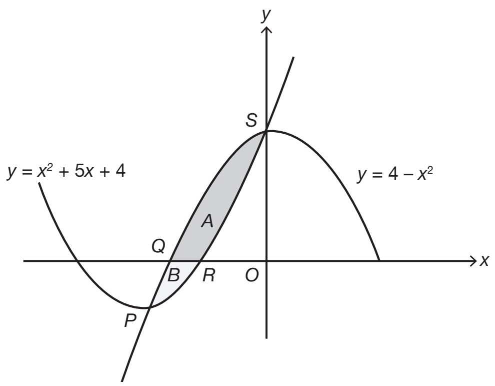
$= 703.808 \text{ cm}^3$

$= 703.808 \text{ m}l$

Isi padu mangkuk tidak cukup untuk menampung isi padu susu 1.5 liter.s

The volume of the bowl is not sufficient to hold the volume of 1.5 litres of milk.

7



$x^2 + 5x + 4 = 4 - x^2$

(a) $2x^2 + 5x = 0$

$x(2x + 5) = 0$

$x = 0, -2.5$

Apabila/*When* $x = 0$, $y = 4 - 0$
$= 4$

Apabila/*When* $x = -2.5$, $y = 4 - (-2.5)^2$
$= -2.25$

$\therefore P(-2.5, -2.25), S(0, 4)$

(b) $Q(x, 0)$, $4 - x^2 = 0$

$x^2 = 4$

$x = \pm 2$

$\therefore Q(-2, 0)$

$R(x, 0)$, $x^2 + 5x + 4 = 0$

$(x + 4)(x + 1) = 0$

$x = -1, -4$

$\therefore R(-1, 0)$

(c) $A = \int_{-2}^{0} 4 - x^2\, dx - \int_{-1}^{0} x^2 + 5x + 4\, dx$

$= \left[4x - \frac{x^3}{3}\right]_{-2}^{0} - \left[\frac{x^3}{3} + \frac{5x^2}{2} + 4x\right]_{-1}^{0}$

$= 0 - \left[4(-2) - \frac{(-2)^3}{3}\right] -$

$\left\{0 - \left[\frac{(-1)^3}{3} + \frac{5(-1)^2}{2} + 4(-1)\right]\right\}$

$= 5\frac{1}{3} - 1\frac{5}{6}$

$= 3\frac{1}{2}$ unit²/*units*²

$B = \left|\int_{-2.5}^{-1} x^2 + 5x + 4\, dx\right| - \left|\int_{-2.5}^{-2} 4 - x^2\, dx\right|$

$= \left|\left[\frac{x^3}{3} + \frac{5x^2}{2} + 4x\right]_{-2.5}^{-1}\right| - \left|\left[4x - \frac{x^3}{3}\right]_{-2.5}^{-2}\right|$

$= \left|\left[\frac{(-1)^3}{3} + \frac{5(-1)^2}{2} + 4(-1)\right]\right.$

$\left. - \left[\frac{(-2.5)^3}{3} + \frac{5(-2.5)^2}{3} + 4(-2.5)\right]\right|$

$- \left|\left[4(-2) - \frac{(-2)^3}{3} - \left[4(-2.5) - \frac{(-2.5)^3}{3}\right]\right]\right|$

$= 2\frac{1}{4} - \frac{13}{24}$

$= 1\frac{13}{24}$ unit²/*units*²

Jumlah luas kawasan berlorek

Total area of shaded region

$= 3\frac{1}{2} + 1\frac{17}{24}$

$= 5\frac{5}{24}$ unit²/*units*²