Jawapan

Praktis 2

Praktis Formatif

1 (a) $\underset{x\to 1}{\text{had}}(x-1)/\lim_{x\to 1}(x-1) = 1-1$
$= 0$

(b) $\underset{x\to 2}{\text{had}}\dfrac{x^2-2}{x}/\lim_{x\to 2}\dfrac{x^2-2}{x} = \dfrac{2^2-2}{2}$
$= 1$

(c) $\underset{x\to 0}{\text{had}}\dfrac{2x-5}{x+3}/\lim_{x\to 0}\dfrac{2x-5}{x+3} = \dfrac{2(0)-5}{(0)+3}$
$= -\dfrac{5}{3}$

(d) $\underset{x\to a}{\text{had}}(x-a)/\lim_{x\to a}(x-a) = a-a$
$= 0$

2 (a) $\underset{x\to 0}{\text{had}}\dfrac{2x^2-5x}{x}/\lim_{x\to 0}\dfrac{2x^2-5x}{x} = \underset{x\to 0}{\text{had}}\,2x-5/\lim_{x\to 0}2x-5$
$= 2(0)-5$
$= -5$

(b) $\underset{x\to 2}{\text{had}}\dfrac{x^2-4}{x-2}/\lim_{x\to 2}\dfrac{x^2-4}{x-2}$
$= \underset{x\to 2}{\text{had}}\dfrac{(x+2)(x-2)}{x-2}/\lim_{x\to 2}\dfrac{(x+2)(x-2)}{x-2}$
$= 2+2$
$= 4$

(c) $\underset{x\to 5}{\text{had}}\dfrac{x^2+4x-45}{x-5}/\lim_{x\to 5}\dfrac{x^2+4x-45}{x-5}$
$= \underset{x\to 5}{\text{had}}\dfrac{(x-5)(x+9)}{x-5}/\lim_{x\to 5}\dfrac{(x-5)(x+9)}{x-5}$
$= \underset{x\to 5}{\text{had}}(x+9)/\lim_{x\to 5}(x+9)$
$= 5+9$
$= 14$

(d) $\underset{x\to 1}{\text{had}}\dfrac{\log_{10}x^2}{\log_{10}x}/\lim_{x\to 1}\dfrac{\log_{10}x^2}{\log_{10}x} = \underset{x\to 1}{\text{had}}\dfrac{2\log_{10}x}{\log_{10}x}/\lim_{x\to 1}\dfrac{2\log_{10}x}{\log_{10}x}$
$= 2$

3 (a) $\underset{x\to 9}{\text{had}}\dfrac{x-9}{\sqrt{x}-3}/\lim_{x\to 9}\dfrac{x-9}{\sqrt{x}-3}$
$= \underset{x\to 9}{\text{had}}\dfrac{(x-9)(\sqrt{x}+3)}{(\sqrt{x}-3)(\sqrt{x}+3)}/\lim_{x\to 9}\dfrac{(x-9)(\sqrt{x}+3)}{(\sqrt{x}-3)(\sqrt{x}+3)}$
$= \underset{x\to 9}{\text{had}}(\sqrt{x}+3)/\lim_{x\to 9}(\sqrt{x}+3)$
$= (\sqrt{9}+3)$
$= 6$

(b) $\underset{x\to -1}{\text{had}}\dfrac{x+1}{\sqrt{x+5}-2}/\lim_{x\to -1}\dfrac{x+1}{\sqrt{x+5}-2}$
$= \underset{x\to -1}{\text{had}}\dfrac{(x+1)(\sqrt{x+5}+2)}{(\sqrt{x+5}-2)(\sqrt{x+5}+2)}$
$\lim_{x\to -1}\dfrac{(x+1)(\sqrt{x+5}+2)}{(\sqrt{x+5}-2)(\sqrt{x+5}+2)}$
$= \underset{x\to -1}{\text{had}}\dfrac{(x+1)(\sqrt{x+5}+2)}{x+5-4}/\lim_{x\to -1}\dfrac{(x+1)(\sqrt{x+5}+2)}{x+5-4}$
$= \underset{x\to -1}{\text{had}}\dfrac{(x+1)(\sqrt{x+5}+2)}{x+1}/\lim_{x\to -1}\dfrac{(x+1)(\sqrt{x+5}+2)}{x+1}$
$= \underset{x\to -1}{\text{had}}\sqrt{x+5}+2/\lim_{x\to -1}\sqrt{x+5}+2$
$= \sqrt{(-1)+5}+2$
$= 4$

(c) $\underset{x\to 9}{\text{had}}\dfrac{\sqrt{x+7}-4}{x-9}/\lim_{x\to 9}\dfrac{\sqrt{x+7}-4}{x-9}$
$= \underset{x\to 9}{\text{had}}\dfrac{(\sqrt{x+7}-4)(\sqrt{x+7}+4)}{(x-9)(\sqrt{x+7}+4)}$
$\lim_{x\to 9}\dfrac{(\sqrt{x+7}-4)(\sqrt{x+7}+4)}{(x-9)(\sqrt{x+7}+4)}$
$= \underset{x\to 9}{\text{had}}\dfrac{x+7-16}{(x-9)(\sqrt{x+7}+4)}/\lim_{x\to 9}\dfrac{x+7-16}{(x-9)(\sqrt{x+7}+4)}$
$= \underset{x\to 9}{\text{had}}\dfrac{x-9}{(x-9)(\sqrt{x+7}+4)}/\lim_{x\to 9}\dfrac{x-9}{(x-9)(\sqrt{x+7}+4)}$
$= \underset{x\to 9}{\text{had}}\dfrac{1}{\sqrt{x+7}+4}/\lim_{x\to 9}\dfrac{1}{\sqrt{x+7}+4}$
$= \dfrac{1}{8}$

(d) $\underset{x\to 2}{\text{had}}\dfrac{\sqrt{6-x}-2}{3-\sqrt{11-x}}/\lim_{x\to 2}\dfrac{\sqrt{6-x}-2}{3-\sqrt{11-x}}$
$= \underset{x\to 2}{\text{had}}\dfrac{(\sqrt{6-x}-2)(3+\sqrt{11-x})}{(3-\sqrt{11-x})(3+\sqrt{11-x})}$
$\lim_{x\to 2}\dfrac{(\sqrt{6-x}-2)(3+\sqrt{11-x})}{(3-\sqrt{11-x})(3+\sqrt{11-x})}$
$= \underset{x\to 2}{\text{had}}\dfrac{(\sqrt{6-x}-2)(3+\sqrt{11-x})}{9-(11-x)}$
$\lim_{x\to 2}\dfrac{(\sqrt{6-x}-2)(3+\sqrt{11-x})}{9-(11-x)}$
$= \underset{x\to 2}{\text{had}}\dfrac{(\sqrt{6-x}-2)(3+\sqrt{11-x})(\sqrt{6-x}+2)}{(x-2)(\sqrt{6-x}+2)}$
$\lim_{x\to 2}\dfrac{(\sqrt{6-x}-2)(3+\sqrt{11-x})(\sqrt{6-x}+2)}{(x-2)(\sqrt{6-x}+2)}$
$= \underset{x\to 2}{\text{had}}\dfrac{(6-x-4)(3+\sqrt{11-x})}{(x-2)(\sqrt{6-x}+2)}$
$\lim_{x\to 2}\dfrac{(6-x-4)(3+\sqrt{11-x})}{(x-2)(\sqrt{6-x}+2)}$

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$= \underset{x \to 2}{\text{had}} \frac{-(x-2)(3+\sqrt{11-x})}{(x-2)(\sqrt{6-x}+2)} / \lim_{x \to 2} \frac{-(x-2)(3+\sqrt{11-x})}{(x-2)(\sqrt{6-x}+2)}$

$= \underset{x \to 2}{\text{had}} \frac{-(3+\sqrt{11-x})}{(\sqrt{6-x}+2)} / \lim_{x \to 2} \frac{-(3+\sqrt{11-x})}{(\sqrt{6-x}+2)}$

$= \frac{-(3+\sqrt{11-2})}{(\sqrt{6-2}+2)}$

$= \frac{-6}{4}$

$= -\frac{3}{2}$

4 (a) $f(4) = 3$

(b) $\underset{x \to 4}{\text{had}} f(x) / \lim_{x \to 4} f(x)$ = tidak wujud/*does not exist*

Had kiri dan had kanan bagi fungsi $f(x)$ adalah tidak sama apabila x menghampiri 4.
The left-hand limit and the right-hand limit of f(x) are different as x approaches 4.

(c) $\underset{x \to 1}{\text{had}} f(x) / \lim_{x \to 1} f(x) = 6$

5 (a)
$$y = 3x + 5$$
$$y + \delta y = 3(x + \delta x) + 5$$
$$= 3x + 3\delta x + 5$$
$$\delta y = 3\delta x$$
$$\frac{\delta y}{\delta x} = 3$$
$$\frac{dy}{dx} \approx \underset{\delta x \to 0}{\text{had}}\ 3 / \lim_{\delta x \to 0} 3$$
$$\frac{dy}{dx} = 3$$

(b)
$$y = x^2 - 7$$
$$y + \delta y = (x + \delta x)^2 - 7$$
$$= x^2 + 2x\delta x + (\delta x)^2 - 7$$
$$\delta y = 2x\delta x + (\delta x)^2$$
$$\frac{\delta y}{\delta x} = 2x + \delta x$$
$$\frac{dy}{dx} \approx \underset{\delta x \to 0}{\text{had}}\ 2x + \delta x / \lim_{\delta x \to 0} 2x + \delta x$$
$$\frac{dy}{dx} = 2x$$

(c)
$$y = x^2 + 2x + 1$$
$$y + \delta y = (x + \delta x)^2 + 2(x + \delta x) + 1$$
$$= x^2 + 2x\delta x + (\delta x)^2 + 2x + 2\delta x + 1$$
$$\delta y = 2x\delta x + (\delta x)^2 + 2\delta x$$
$$\frac{\delta y}{\delta x} = 2x + \delta x + 2$$
$$\frac{dy}{dx} \approx \underset{\delta x \to 0}{\text{had}}\ 2x + \delta x + 2 / \lim_{\delta x \to 0} 2x + \delta x + 2$$
$$\frac{dy}{dx} = 2x + 2$$

(d)
$$y = -x^3 + 9$$
$$y + \delta y = -(x + \delta x)^3 + 9$$
$$= -(x + \delta x)[x^2 + 2x\delta x + (\delta x)^2] + 9$$
$$= -[x^3 + 3x^2\,\delta x + 3x(\delta x)^2 + (\delta x)^3] + 9$$
$$\delta y = -[3x^2\,\delta x + 3x\,(\delta x)^2 + (\delta x)^3]$$
$$\frac{\delta y}{\delta x} = -[3x^2 + 3x(\delta x) + (\delta x)^2]$$
$$\frac{dy}{dx} \approx \underset{\delta x \to 0}{\text{had}} -[3x^2 + 3x(\delta x) + (\delta x)^2]$$
$$\lim_{\delta x \to 0} -[3x^2 + 3x(\delta x) + (\delta x)^2]$$
$$\frac{dy}{dx} = -3x^2$$

(e)
$$y = 2 - \frac{3}{x}$$
$$y + \delta y = 2 - \frac{3}{x + \delta x}$$
$$\delta y = 2 - \frac{3}{x + \delta x} - \left(2 - \frac{3}{x}\right)$$
$$= -\frac{3}{x + \delta x} + \frac{3}{x}$$
$$= \frac{-3x + 3x + 3\delta x}{x(x + \delta x)}$$
$$= \frac{3\delta x}{x^2 + x\delta x}$$
$$\frac{\delta y}{\delta x} = \frac{3}{x^2 + x\delta x}$$
$$\frac{dy}{dx} \approx \underset{\delta x \to 0}{\text{had}} \frac{3}{x^2 + x\delta x} / \lim_{\delta x \to 0} \frac{3}{x^2 + x\delta x}$$
$$\frac{dy}{dx} = \frac{3}{x^2}$$

6 (a)
$$y = x^2 - ax + b$$
$$y + \delta y = (x + \delta x)^2 - a(x + \delta x) + b$$
$$= x^2 + 2x\delta x + (\delta x)^2 - ax - a\delta x + b$$
$$\delta y = 2x\delta x + (\delta x)^2 - a\delta x$$
$$\frac{\delta y}{\delta x} = 2x + \delta x - a$$
$$\frac{dy}{dx} \approx \underset{\delta x \to 0}{\text{had}}\ 2x + \delta x - a / \lim_{\delta x \to 0} 2x + \delta x - a$$
$$\frac{dy}{dx} = 2x - a$$

(b) $\frac{dy}{dx} = 2$ pada/*at* $(2, -3)$

$2(2) - a = 2$

$a = 2$

Gantikan $(2, -3)$ ke dalam $y = x^2 - 2x + b$,
Substitute $(2, -3)$ *into* $y = x^2 - 2x + b$,

$-3 = (2)^2 - 2(2) + b$

$b = -3$

7 (a) $\frac{dy}{dx} = 12x$

(b) $\frac{dy}{dx} = -4x^3$

(c)
$$y = -x^{\frac{4}{3}}$$
$$\frac{dy}{dx} = -\frac{4x^{\frac{1}{3}}}{3}$$
$$= -\frac{4}{3}x^{\frac{1}{3}}$$

(d)
$$y = -2x^{-2}$$
$$\frac{dy}{dx} = 4x^{-3}$$

8 (a) $\frac{d}{dx}(2x^2 + 3x - 9) = 4x + 3$

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(b) $\frac{d}{dx}(x^2 + \frac{2}{x}) = \frac{d}{dx}(x^2 + 2x^{-1})$
$= 2x - 2x^{-2}$
$= 2x - \frac{2}{x^2}$

(c) $\frac{d}{dx}\left(5x^3 + 2x^2 + 4x - 7 - \frac{1}{x} + \frac{3}{x^2}\right)$
$= \frac{d}{dx}(5x^3 + 2x^2 + 4x - 7 - x^{-1} + 3x^{-2})$
$= 15x^2 + 4x + 4 + x^{-2} - 6x^{-3}$
$= 15x^2 + 4x + 4 + \frac{1}{x^2} - \frac{6}{x^3}$

9 (a) $f(x) = \left(\frac{1}{2}x^5 - x^3 - 5x^2\right)$
$f'(x) = \frac{5}{2}x^4 - 3x^2 - 10x$

(b) $f(x) = x^3 + 3x^2 - 5x - 15$
$f'(x) = 3x^2 + 6x - 5$

(c) $f(x) = x^2 - 1 + 4x^{-1}$
$f'(x) = 2x - 4x^{-2}$

(d) $f(x) = \frac{(x-2)(x+1)}{(x-2)}$
$= x + 1$
$f'(x) = 1$

10 (a) $f'(x) = 4(3x - 5)^3(3)$
$= 12(3x - 5)^3$

(b) $f'(x) = 15(x^3 + 4x)^2(3x^2 + 4)$
$= 15(3x^2 + 4)(x^3 + 4x)^2$

(c) $f(x) = 2(5x^2 - 3x)^{-10}$
$f'(x) = -20(5x^2 - 3x)^{-11}(10x - 3)$
$= \frac{-20(10x - 3)}{(5x^2 - 3x)^{11}}$

11 (a) $y = 6x^2\,[x(1 + 5x)]^3$
$= 6x^2(x^3)(1 + 5x)^3$
$= 6x^5\,(1 + 5x)^3$
$\frac{dy}{dx} = (1 + 5x)^3\,\frac{d}{dx}(6x^5) + (6x^5)\,\frac{d}{dx}(1 + 5x)^3$
$= (1 + 5x)^3\,(30x^4) + (6x^5)(3)(1 + 5x)^2\,(5)$
$= 30x^4\,(1 + 5x)^2\,[1 + 5x + 3x]$
$= 30x^4\,(1 + 5x)^2\,(1 + 8x)$

(b) $\frac{dy}{dx} = (7x + 3)^5\,\frac{d}{dx}(x) + (x)\,\frac{d}{dx}(7x + 3)^5$
$= (7x + 3)^5 + x(5)(7x + 3)^4\,(7)$
$= (7x + 3)^4\,[7x + 3 + 35x]$
$= (42x + 3)(7x + 3)^4$

(c) $\frac{dy}{dx} = (1 - 2x^2)^{10}\,\frac{d}{dx}(4x^2 - 3x) + (4x^2 - 3x)\,\frac{d}{dx}(1 - 2x^2)^{10}$
$= (1 - 2x^2)^{10}\,(8x - 3) + (4x^2 - 3x)(10)(1 - 2x^2)^9\,(-4x)$
$= (1 - 2x^2)^9\,[(1 - 2x^2)\,(8x - 3) - 40x\,(4x^2 - 3x)]$
$= (1 - 2x^2)^9\,[8x - 3 - 16x^3 + 6\,x^2 - 160x^3 + 120x^2]$
$= (1 - 2x^2)^9\,(8x - 3 - 176x^3 + 126x^2)$

12 (a) $\frac{dy}{dx} = \frac{(2x + 1)\,\frac{d}{dx}(x - 2) - (x - 2)\frac{d}{dx}(2x + 1)}{(2x + 1)^2}$
$= \frac{(2x + 1)(1) - (x - 2)(2)}{(2x + 1)^2}$
$= \frac{2x + 1 - 2x + 4}{(2x + 1)^2}$
$= \frac{5}{(2x + 1)^2}$

(b) $\frac{dy}{dx} = \frac{(x - 1)\frac{d}{dx}(x^2 + 3x - 4) - (x^2 + 3x - 4)\frac{d}{dx}(x - 1)}{(x - 1)^2}$
$= \frac{(x - 1)(2x + 3) - (x^2 + 3x - 4)(1)}{(x - 1)^2}$
$= \frac{2x^2 + x - 3 - x^2 - 3x + 4}{(x - 1)^2}$
$= \frac{x^2 - 2x + 1}{(x - 1)^2}$
$= \frac{(x - 1)^2}{(x - 1)^2}$
$= 1$

(c) $\frac{dy}{dx} = \frac{(2x - 1)^2\frac{d}{dx}(x^3) - (x^3)\frac{d}{dx}(2x - 1)^2}{[(2x - 1)^2]^2}$
$= \frac{(2x - 1)^2(3x^2) - (x^3)(2)(2x - 1)(2)}{(2x - 1)^4}$
$= \frac{x^2(2x - 1)[3(2x - 1) - (4x)]}{(2x - 1)^4}$
$= \frac{x^2(6x - 3 - 4x)}{(2x - 1)^3}$
$= \frac{x^2(2x - 3)}{(2x - 1)^3}$

13 $m_t = \frac{dy}{dx}$
$= (4x + 1)\frac{d}{dx}\left(x^{\frac{1}{2}}\right) + \left(x^{\frac{1}{2}}\right)\frac{d}{dx}(4x + 1)$
$= (4x + 1)\left(\frac{1}{2}\right)\left(x^{-\frac{1}{2}}\right) + (x^{\frac{1}{2}})(4)$

Pada/*At* $x = 4$,
$m_t = [4(4) + 1]\left(\frac{1}{2}\right)(4)^{-\frac{1}{2}} + (4^{\frac{1}{2}})(4)$
$= \frac{49}{4}$

14 $y = 5x^{-2}$
$\frac{dy}{dx} = -10x^{-3}$

Pada/*At* $x = 2$,
$\frac{dy}{dx} = -\frac{10}{8}$
$= -\frac{5}{4}$

15 $y = 5x^m$
$\frac{dy}{dx} = 5m\,x^{m-1}$

Bandingkan dengan/*Compare with* $\frac{dy}{dx} = x^n$

$5m = 1$, $n = m - 1$
$m = \frac{1}{5}$ $= \frac{1}{5} - 1$
$= -\frac{4}{5}$

16 $f'(x) = 3ax^2 - 2bx + 9$
$= 3a\left(x^2 - \frac{2b}{3a}x\right) + 9$
$= 3a\left[\left(x - \frac{b}{3a}\right)^2 - \frac{b^2}{9a^2}\right] + 9$

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$= 3a\left(x - \frac{b}{3a}\right)^2 - \frac{b^2}{3a} + 9$

$f'(x)$ sentiasa positif apabila/$f'(x)$ *is always positive when*

$-\frac{b^2}{3a} + 9 > 0$

$\frac{b^2}{3a} < 9$

$b^2 < 27a$ (tertunjuk/*shown*)

atau/*or*

$f'(x) = 3ax^2 - 2bx + 9$

$f'(x)$ sentiasa positif apabila/$f'(x)$ *is always positive when*

$(-2b)^2 - 4(3a)(9) < 0$ (tiada punca/*no root*)

$4b^2 - 4(27a) < 0$

$b^2 - (27a) < 0$

$b^2 < 27a$ (tertunjuk/*shown*)

17 $\frac{d}{dx}(ax^m + bx^n) = 12x^s + 9x^t$

$am(x^{m-1}) + bn(x^{n-1}) = 12x^s + 9x^t$

$am = 12, bn = 9, s = m - 1, t = n - 1$

(a) $\frac{s}{t} = \frac{m-1}{n-1}$

$= \frac{\frac{12}{a} - 1}{\frac{9}{b} - 1}$

$= \frac{\frac{12-a}{a}}{\frac{9-b}{b}}$

$= \frac{b(12-a)}{a(9-b)}$

(b) $\frac{s}{t} = \frac{5}{3}$ $\qquad$ $\frac{m}{n} = \frac{3}{2}$

$\frac{b(12-a)}{a(9-b)} = \frac{5}{3}$ $\qquad$ $2m = 3n$

$36b - 3ab = 45a - 5ab$ $\qquad$ $2\left(\frac{12}{a}\right) = 3\left(\frac{9}{b}\right)$

$36b + 2ab = 45a$ … ① $\qquad$ $b = \frac{9}{8}a$ … ②

Gantikan ② ke dalam ①/*Substitute* ② *into* ①,

$36\left(\frac{9}{8}a\right) + 2a\left(\frac{9}{8}a\right) = 45a$

$36a + 2a^2 - 40a = 0$

$2a^2 - 4a = 0$

$2a(a - 2) = 0$

$a = 2 (a > 0)$

$b = \frac{9}{8}(2)$

$= \frac{9}{4}$

(c) $m = \frac{12}{2}$ $\qquad$ $s = 6 - 1$

$= 6$ $\qquad$ $= 5$

$n = 9 \div \frac{9}{4}$ $\qquad$ $t = 4 - 1$

$= 4$ $\qquad$ $= 3$

18 (a) $\frac{dy}{dx} = 12x^2 - 7x^{-2}$

$\frac{d^2y}{dx^2} = 24x + 14x^{-3}$

(b) $\frac{dy}{dx} = 5(2x^3 - 3)^4(6x^2)$

$= 30x^2(2x^3 - 3)^4$

$\frac{d^2y}{dx^2} = (2x^3 - 3)^4\frac{d}{dx}(30x^2) + (30x^2)\frac{d}{dx}(2x^3 - 3)^4$

$= (2x^3 - 3)^4(60x) + 30x^2(4)(2x^3 - 3)^3(6x^2)$

$= 60x(2x^3 - 3)^3[2x^3 - 3 + 12x^3)$

$= 60x(2x^3 - 3)^3(14x^3 - 3)$

(c) $\frac{dy}{dx} = 4\pi x^2$

$\frac{d^2y}{dx^2} = 8\pi x$

(e) $\frac{dy}{dx} = 3(-2)(x^2 + 1)^{-3}(2x)$

$= \frac{-12x}{(x^2 + 1)^3}$

$\frac{d^2y}{dx^2} = \frac{(x^2 + 1)^3\frac{d}{dx}(-12x) - (-12x)\frac{d}{dx}(x^2 + 1)^3}{[(x^2 + 1)^3]^2}$

$= \frac{(x^2 + 1)^3(-12) - (-12x)(3)(x^2 + 1)^2(2x)}{(x^2 + 1)^6}$

$= \frac{(-12)(x^2 + 1)^2[x^2 + 1 - 6x^2]}{(x^2 + 1)^6}$

$= \frac{-12(1 - 5x^2)}{(x^2 + 1)^4}$

19 $\frac{dy}{dx} = 12x^2 - 4x$ $\qquad$ $\frac{d^2y}{dx^2} = 24x - 4$

Apabila/*When* $x = 2$, $\qquad$ Apabila/*When* $x = 2$,

$\frac{dy}{dx} = 12(2)^2 - 4(2)$ $\qquad$ $\frac{d^2y}{dx^2} = 24(2) - 4$

$= 40$ $\qquad$ $= 44$

20 $y = x^{-1}$, $\frac{dy}{dx} = -x^{-2}$, $\qquad$ $\frac{d^2y}{dx^2} = 2x^{-3}$

$y + \frac{d^2y}{dx^2} = \frac{1}{x} + \frac{2}{x^3}$

$= \frac{x^2 + 2}{x^3}$

$= \left(\frac{1}{x}\right)^3(x^2 + 2)$

$= y^3(x^2 + 2)$ (terbukti/*proven*)

21 $\frac{d}{dx}\left(\frac{x^4}{12} - x^3 + \frac{9}{2}x^2 + 6x - 3\right) = \frac{x^3}{3} - 3x^2 + 9x + 6$

$\frac{d}{dx}\left(\frac{x^3}{3} - 3x^2 + 9x + 6\right) = x^2 - 6x + 9$

$\frac{d^2}{dx^2}\left(\frac{x^4}{12} - x^3 + \frac{9}{2}x^2 + 6x - 3\right)$

$= x^2 - 6x + 9$

$= (x - 3)^2$

Terbukti bahawa sentiasa positif bagi semua nilai x

Proven that always positive for all the values of x

22 $h'(x) = 9x^2 + 2mx + 1$

$h''(x) = 18x + 2m$

$h''(1) = 10$

$18(1) + 2m = 10$

$2m = -8$

$m = -4$

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23 $y = x^2 - 5x + 9, \frac{dy}{dx} = 2x - 5, \frac{d^2y}{dx^2} = 2$

$$(2x-5)^2 - 2 + 2(x^2 - 5x + 9) = 3x + 5$$
$$4x^2 - 20x + 25 - 2 + 2x^2 - 10x + 18 - 3x - 5 = 0$$
$$6x^2 - 33x + 36 = 0$$
$$2x^2 - 11x + 12 = 0$$
$$(2x-3)(x-4) = 0$$
$$x = \frac{3}{2}, 4$$

24 (a) $f'(x) = 9x^2 - 4x - 5$

(b) (i) $m_A = 9(-1)^2 - 4(-1) - 5$
$= 8$
$m_B = 9(0)^2 - 4(0) - 5$
$= -5$
$m_C = 9(1)^2 - 4(1) - 5$
$= 0$

(ii) Kecerunan tangen pada titik A adalah positif dan garis tangen condong ke kanan.
The gradient of tangent at point A is positive and the tangent slants to the right.
Kecerunan tangen pada titik B adalah negatif dan garis tangen condong ke kiri.
The gradient of tangent at point B is negative and the tangent slants to the left.
Kecerunan tangen pada titik C adalah sifar dan garis tangen merupakan suatu garis mengufuk.
The gradient of tangent at point C is zero and the tangent is a horizontal line.

25 (a) $m_t = \frac{dy}{dx}$
$= 2(2x^2+3)(4x)$
$m_{t_{x=1}} = 8(1)[2(1)^2+3]$
$= 40$

(b) $m_t = \dfrac{(3-2x)\frac{d}{dx}(4-3x^2) - (4-3x^2)\frac{d}{dx}(3-2x)}{(3-2x)^2}$
$= \dfrac{(3-2x)(-6x) - (4-3x^2)(-2)}{(3-2x)^2}$
$m_{t_{x=2}} = \dfrac{[(3-2(2)](-6)(2) - (4-3(2)^2)(-2)}{[3-2(2)]^2}$
$= -4$

26 (a) $m_t = \frac{dy}{dx}$
$= 6x^2 - 6x$
$m_{t_{x=1}} = 6(1)^2 - 6(1)$
$= 0$

(b) $m_t = \frac{dy}{dx}$
$8 = x^2 + 2x$
$x^2 + 2x - 8 = 0$
$(x+4)(x-2) = 0$
$x = -4, 2$
$y = \frac{(-4)^3}{3} + (-4)^2 - 1$ atau/*or* $y = \frac{2^3}{3} + 2^2 - 1$
$= -\frac{19}{3}$ $= \frac{17}{3}$
Koordinat/*Coordinates*: $\left(-4, -\frac{19}{3}\right), \left(2, \frac{17}{3}\right)$

(c) $\frac{dy}{dx} = 2ax + b$
$m_{t_{x=2}} = 5$ $m_{t_{x=-3}} = 0$
$4a + b = 5 \ldots$ ① $-6a + b = 0 \ldots$ ②
① – ②: $10a = 5$
$a = \frac{1}{2}$
$b = 5 - 4\left(\frac{1}{2}\right)$
$= 3$

27 $m_t = \frac{dy}{dx}$
$= -2 - 2x$

(a) Pada/*At* $A(1, 5)$,
$m_t = -2 - 2(1)$
$= -4$
$m_n = \frac{1}{4}$
Persamaan tangen/*Equation of tangent*:
$y - 5 = (-4)(x-1)$
$y = -4x + 4 + 5$
$y = -4x + 9$

Persamaan normal/*Equation of normal*:
$y - 5 = \left(\frac{1}{4}\right)(x-1)$
$4y - 20 = x - 1$
$x - 4y + 19 = 0$

(b) Pada/*At* $C(-1, 9)$,
$m_t = -2 - 2(-1)$
$= 0 \rightarrow$ garis mengufuk/*horizontal line*
Persamaan tangen/*Equation of tangent*: $y = 9$
Persamaan normal/*Equation of normal*: $x = -1$

28 (a) $m_t = \frac{dy}{dx}$
$= 6x + 8$
Pada titik/*At point* $(-2, 6)$, $m_t = 6(-2) + 8$
$= -4$
$m_n = \frac{1}{4}$
Persamaan normal/*Equation of normal*:
$y - 6 = \left(\frac{1}{4}\right)[x - (-2)]$
$4y - 24 = x + 2$
$x - 4y + 26 = 0$

(b) $m_t = \frac{dy}{dx}$
$= 2ax + b$
$m_{t_{x=4}} = 2a(4) + b$
$= 8a + b$
$m_{AB} = \frac{0-1}{12-4}$
$= -\frac{1}{8}$
$m_{t_{x=4}} \times m_{AB} = -1$
$8a + b = 8 \ldots$ ①

$y = ax^2 + bx$
Pada/*At* $P(4, 8)$,
$8 = a(4)^2 + b(4)$
$8 = 16a + 4b$
$4a + b = 2 \ldots$ ②

① – ②: $4a = 6$
$a = \frac{3}{2}$

Daripada/*From* ①: $b = 8 - 8\left(\frac{3}{2}\right)$
$= -4$

29 (a) $m_t = 0$

$10x - 2 = 0$

$x = \frac{1}{5}$

$y = 5\left(\frac{1}{5}\right)^2 - 2\left(\frac{1}{5}\right) + 1$

$= \frac{4}{5}$

Titik pusingan/*Turning point* $= \left(\frac{1}{5}, \frac{4}{5}\right)$

(i)

x	0	$\frac{1}{5}$	$\frac{2}{5}$
$\frac{dy}{dx}$	−2	0	2
Lakaran tangen *Sketch of the tangent*			
Lakaran graf *Sketch of the graph*			

$\therefore \left(\frac{1}{5}, \frac{4}{5}\right)$ ialah titik minimum/*is a minimum point*

(ii) $\frac{dy}{dx} = 10x - 2$

$\frac{d^2y}{dx^2} = 10 > 0$

$\therefore \left(\frac{1}{5}, \frac{4}{5}\right)$ ialah titik minimum/*is a minimum point*

(b) $y = \frac{x^2}{x + 1}$

$$\frac{dy}{dx} = \frac{(x+1)\frac{d}{dx}(x^2) - (x^2)\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$= \frac{(x+1)(2x) - (x^2)(1)}{(x+1)^2}$$

$$= \frac{2x^2 + 2x - x^2}{(x+1)^2}$$

$$= \frac{x^2 + 2x}{(x+1)^2}$$

$$\frac{x^2 + 2x}{(x+1)^2} = 0$$

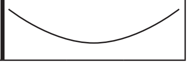
$x + 1 \neq 0, x(x + 2) = 0$

$x = 0$ atau/*or* $x = -2$

$y = \frac{(0)^2}{(0) + 1} = 0$ $\quad y = \frac{(-2)^2}{(-2) + 1} = -4$

Titik pusingan/*Turning points* = (0, 0), (−2, −4)

(i)

x	−3	−2	−1.5	−0.5	0	1
$\frac{dy}{dx}$	0.75	0	−3	−3	0	0.75
Lakaran tangen *Sketch of the tangent*						
Lakaran graf *Sketch of the graph*						

$\therefore$ (−2, −4) ialah titik maksimum
(−2, −4) *is a maximum point*
(0, 0) ialah titik minimum/*is a minimum point*

(ii) $\frac{dy}{dx} = \frac{x^2 + 2x}{(x+1)^2}$

$$\frac{d^2y}{dx^2} = \frac{(x+1)^2\frac{d}{dx}(x^2 + 2x) - (x^2 + 2x)\frac{d}{dx}(x+1)^2}{[(x+1)^2]^2}$$

$$= \frac{(x+1)^2(2x+2) - (x^2+2x)(2)(x+1)(1)}{(x+1)^4}$$

Apabila/*When* $x = 0$,

$$\frac{d^2y}{dx^2} = \frac{(0+1)^2(0+2) - (0+0)(2)(0+1)(1)}{(0+1)^4}$$

$= 2 > 0$

$\therefore$ (0, 0) ialah titik minimum/*is a minimum point*

Apabila/*When* $x = -2$,

$$\frac{d^2y}{dx^2} = \frac{(-2+1)^2(-4+2) - ((-2)^2 + 2(-2))(2)(-2+1)(1)}{(-2+1)^4}$$

$= -2 < 0$

$\therefore$ (−2, −4) ialah titik maksimum
(−2, −4) *is a maximum point*

$= -\frac{2}{3} < 0$

(c) $y = 7 - x^3$

$\frac{dy}{dx} = -3x^2$

$-3x^2 = 0$

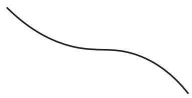
$x = 0$

$y = 7 - (0)^3$

$= 7$

Titik pusingan/*Turning point* = (0, 7)

(i)

x	− 1	0	1
$\frac{dy}{dx}$	− 3	0	− 3
Lakaran tangen *Sketch of the tangent*			
Lakaran graf *Sketch of the graph*			

$\therefore$ (0, 7) ialah titik lengkok balas
(0, 7) *is an inflection point*

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(ii) $\frac{dy}{dx} = -3x^2$

$\frac{d^2y}{dx^2} = -6x$

Apabila/*When* $x = 0$, $\frac{d^2y}{dx^2} = 0$

$\therefore$ (0, 7) ialah titik lengkok balas
(0, 7) *is an inflection point*

30 (a) $y = x(x^2 - 6x + 9)$

$= x^3 - 6x^2 + 9x$

$\frac{dy}{dx} = 3x^2 - 12x + 9$

$3x^2 - 12x + 9 = 0$

$x^2 - 4x + 3 = 0$

$(x - 1)(x - 3) = 0$

$x = 1$

$y = (1)^3 - 6(1)^2 + 9(1)$

$= 4$

atau/*or*

$x = 3$

$y = (3)^3 - 6(3)^2 + 9(3)$

$= 0$

Titik pusingan/*Turning points* = (1, 4), (3, 0)

$\frac{d^2y}{dx^2} = 6x - 12$

Apabila/*When* $x = 1$, $\frac{d^2y}{dx^2} = 6(1) - 12$

$= -6 < 0$

$\therefore$ (1, 4) ialah titik maksimum/*is a maximum point*

Apabila/*When* $x = 3$, $\frac{d^2y}{dx^2} = 6(3) - 12$

$= 6 > 0$

$\therefore$ (3, 0) ialah titik minimum/*is a minimum point*

(b) $b = \sqrt{40 - h^2}$

$A = h(40 - h^2)^{\frac{1}{2}}$

Titik pegun wujud apabila
Stationary point exists when

$\frac{dA}{dx} = 0$

$(40 - h^2)^{\frac{1}{2}}\frac{d}{dh}(h) + (h)\frac{d}{dh}(40 - h^2)^{\frac{1}{2}} = 0$

$(40 - h^2)^{\frac{1}{2}} + \frac{h(-2h)}{2(40 - h^2)^{\frac{1}{2}}} = 0$

$\frac{40 - h^2 - h^2}{(40 - h^2)^{\frac{1}{2}}} = 0$

$(40 - h^2)^{\frac{1}{2}} \neq 0$, $40 - 2h^2 = 0$

$2h^2 = 40$

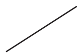
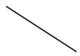
$h = \sqrt{20} \, (h > 0)$

$b = \sqrt{40 - \sqrt{20}^2}$

$= \sqrt{20}$

$A = \sqrt{20} \times \sqrt{20}$

$= 20$

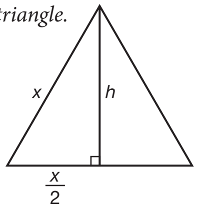
h	4	$\sqrt{20}$	5
$\frac{dA}{dx}$	1.63	0	−2.58
Lakaran tangen *Sketch of the tangent*			
Lakaran graf *Sketch of the graph*			

$\therefore A = 20$ ialah nilai maksimum/*is a maximum value*

(c) $3x + 4y = 120$

$y = \frac{120 - 3x}{4}$

Biar h mewakili tinggi segi tiga itu.
Let h represents the height of the triangle.



$h = \sqrt{x^2 - \left(\frac{x}{2}\right)^2}$

$= \frac{\sqrt{3}x}{2}$

$A = A_\Delta + A_\square$

$= \frac{1}{2}(x)\left(\frac{\sqrt{3}x}{2}\right) + y^2$

$= \frac{\sqrt{3}x^2}{4} + \left(\frac{120 - 3x}{4}\right)^2$

$= \frac{4\sqrt{3}x^2 + [3(40 - x)]^2}{16}$

$= \frac{4\sqrt{3}x^2 + 9(40 - x)^2}{16}$

$= \frac{9(40 - x)^2 + 4\sqrt{3}x^2}{16}$ (tertunjuk/*shown*)

A pegun apabila/*A is stationary when*

$\frac{dA}{dx} = 0$

$\frac{18(40 - x)(-1) + 8\sqrt{3}x}{16} = 0$

$\frac{18x - 720 + 8\sqrt{3}x}{16} = 0$

$18x - 720 + 8\sqrt{3}x = 0$

$(18 + 8\sqrt{3})x = 720$

$x = 22.6$

$\frac{d^2A}{dx^2} = \frac{18 + 8\sqrt{3}}{16}$

$= 1.991 > 0$

$\therefore A$ adalah minimum apabila/*A is minimum when* $x = 22.6$

(d) Panjang/*Length*, $y = (50 - x)$ cm

$A = xy$

$= x(50 - x)$

$= 50x - x^2$

A adalah maksimum apabila/*A is maximum when*

$\frac{dA}{dx} = 0$

$50 - 2x = 0$

$x = 25$

$y = 50 - 25$

$= 25$

$\because x = y$, $\therefore$ A adalah maksimum apabila bentuk itu ialah sebuah segi empat sama.
A is maximum when the shape is a square.

31 (a) $\frac{dV}{dt} = 2 \text{ cm}^3\text{s}^{-1}$, $V = 729 \text{ cm}^3$

$x^3 = 729$

$x = 9$

$V = x^3$

$\frac{dV}{dx} = 3x^2$

$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt}$

$\frac{dx}{dt} = \frac{1}{3(9)^2} \times 2$

$= \frac{2}{243} \text{ cms}^{-1}$

(b) $\frac{dA}{dt} = 6 \text{ cm}^2\text{s}^{-1}$, $r = 5$ cm

(i) $A = 4\pi r^2$

$\frac{dA}{dr} = 8\pi r$

$\frac{dr}{dt} = \frac{dr}{dA} \times \frac{dA}{dt}$

$\frac{dr}{dt} = \frac{1}{8\pi(5)} \times 6$

$= \frac{3}{20\pi} \text{ cms}^{-1}$

(ii) $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$\frac{dV}{dt} = \frac{dV}{dA} \times \frac{dA}{dt}$

$= \frac{dV}{dr} \times \frac{dr}{dA} \times \frac{dA}{dt}$

$\frac{dV}{dr} = 4\pi(5)^2 \times \frac{3}{20\pi}$

$= 15 \text{ cm}^3\text{s}^{-1}$

(c) $\frac{dV}{dt} = 80 \text{ cm}^3\text{s}^{-1}$, $V = \frac{1}{3}\pi x^3 \text{ cm}^3$, $x = 10$ cm

(i) $V = \frac{1}{3}\pi x^3$

$\frac{dV}{dx} = \pi x^2$

$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt}$

$\frac{dx}{dt} = \frac{1}{\pi(10)^2} \times 80$

$= \frac{4}{5\pi} \text{ cms}^{-1}$

(ii) $A = \pi r^2$, $r = x \Rightarrow A = \pi x^2$

$\frac{dA}{dx} = 2\pi r$

$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$

$= 2\pi(10) \times \frac{4}{5\pi}$

$= 16 \text{ cm}^2\text{s}^{-1}$

32 (a) $\frac{dy}{dx} = 6x^2 - 10x + 1$

Apabila/*When* $x = 1$, $\frac{dy}{dx} = 6(1)^2 - 10(1) + 1$

$= -3$

$\delta x = 0.02$

$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$

$\delta y \approx \frac{dy}{dx} \times \delta x$

$= -3 \times 0.02$

$\delta y = -0.06$

(b) $\frac{dy}{dx} = \frac{-36}{(2x-5)^3}$

(i) Apabila/*When* $x = 3$,

$\delta x = p$

$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$

$\delta y = \frac{-36}{[2(3)-5]^3} \times p$

$= -36p$

(ii) Apabila/*When* $y = 1$,

$\frac{9}{(2x-5)^2} = 1$

$(2x-5)^2 = 9$

$2x - 5 = \pm 3$

$x = 1, 4$

$\delta y = -p$

$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$

$\frac{-p}{\delta x} \approx \frac{-36}{[2(1)-5]^3}$ atau/*or* $\frac{-p}{\delta x} \approx \frac{-36}{[2(4)-5]^3}$

$\frac{-p}{\delta x} \approx \frac{4}{3}$ $\qquad$ $\frac{-p}{\delta x} \approx -\frac{4}{3}$

$\delta x \approx \pm\frac{3}{4}p$

(c) $\frac{dy}{dx} = 4x^3$

(i) $x = 2$, $\delta x = 0.03$

$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$

$\delta y \approx 4(2)^3 \times 0.03$

$= 0.96$

$y_n = y_o + \delta y$

$2.03^4 = 2^4 + 0.96$

$= 16.96$

(ii) $x = 2$, $\delta x = -0.01$

$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$

$\delta y \approx 4(2)^3 \times (-0.01)$

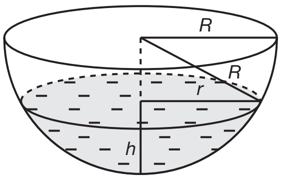
$= -0.32$

$y_n = y_o + \delta y$

$1.99^4 = 2^4 - 0.32$

$= 15.68$

33 (a)



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$$
\begin{aligned}
r^2 + (R-h)^2 &= R^2 \\
r^2 &= R^2 - (R^2 - 2Rh + h^2) \\
&= R^2 - R^2 + 2Rh - h^2 \\
&= 2Rh - h^2 \\
r &= \sqrt{2Rh - h^2} \text{ (tertunjuk/\textit{shown})}
\end{aligned}
$$

(b) $\dfrac{dV}{dt} = 300 \text{ cm}^3\text{min}^{-1}$

$A = \pi r^2$

$\quad = \pi(2Rh - h^2)$

Apabila/*When* $2h = R$,

$A = \pi\left[R^2 - \left(\dfrac{R}{2}\right)^2\right]$

$\quad = \dfrac{3\pi R^2}{4}$

$\dfrac{dA}{dR} = \dfrac{3\pi R}{2}$

$V = \dfrac{\pi}{3}(3Rh^2 - h^3)$

Apabila/*When* $2h = R$,

$V = \dfrac{\pi}{3}\left[3R\left(\dfrac{R}{2}\right)^2 - \left(\dfrac{R}{2}\right)^3\right]$

$\quad = \dfrac{\pi}{3}\left(\dfrac{3R^3}{4} - \dfrac{R^3}{8}\right)$

$\quad = \dfrac{5\pi R^3}{24}$

$\dfrac{dV}{dR} = \dfrac{5\pi R^2}{8}$

$\dfrac{dA}{dt} = \dfrac{dA}{dR} \times \dfrac{dR}{dV} \times \dfrac{dV}{dt}$

$\quad = \dfrac{3\pi R}{2} \times \dfrac{8}{5\pi R^2} \times 300$

$\quad = \dfrac{720}{R} \text{ cm}^2\text{min}^{-1}$

Praktis Sumatif

Kertas 1

1 $\delta y = 4x\,\delta x + 2(\delta x)^2 + 3\delta x$

$\dfrac{\delta y}{\delta x} = 4x + 2\delta x + 3$

$\dfrac{dy}{dx} \approx \underset{\delta x \to 0}{\text{had}}\, 4x + 2\delta x + 3 / \lim_{\delta x \to 0} 4x + 2\delta x + 33$

$\quad = 4x + 3$

Apabila/*When* $x = 2$,

$\dfrac{dy}{dx} = 4(2) + 3$

$\quad = 11$

2 $\dfrac{d}{dx}\left(\dfrac{x^3}{3 - x^3}\right) = \dfrac{(3 - x^3)\dfrac{d}{dx}(x^3) - (x^3)\dfrac{d}{dx}(3 - x^3)}{(3 - x^3)^2}$

$\quad = \dfrac{(3 - x^3)(3x^2) - (x^3)(-3x^2)}{(3 - x^3)^2}$

$\quad = \dfrac{3x^2(3 - x^3 + x^3)}{(3 - x^3)^2}$

$\quad = \dfrac{9x^2}{(3 - x^3)^2}$

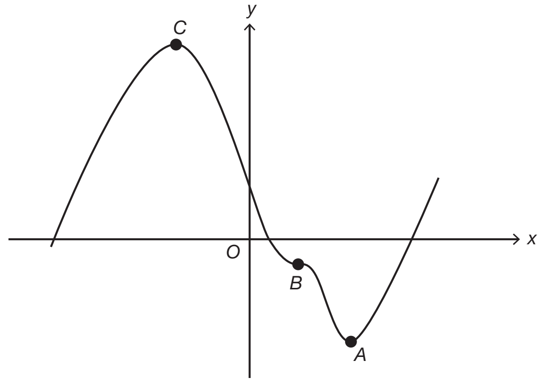
Bandingkan dengan/*Compare with* $\dfrac{kx^m}{(3 - x^3)^n}$,

$k = 9, m = 2, n = 2$

3 $f'(x_a) = f'(x_b) = f'(x_c) = 0 \rightarrow A, B, C$ ialah titik pusingan
A, B, C are turning points

$f''(x_b) = 0 \rightarrow B$ ialah titik lengkok balas
B is an inflection point

$f''(x_c) < f''(x_b) < f''(x_a) \rightarrow C$ ialah titik maksimum dan A ialah titik minimum.
C is a maximum point and A is a minimum point.



(Terima semua graf yang berbentuk serupa seperti di atas tanpa mengambil kira kedudukan graf.)
(Accept all graphs that have the similar shape regardless its position.)

4 (a) $\underset{x \to 3}{\text{had}} \dfrac{x - 3}{4 - \sqrt{19 - x}} / \lim_{x \to 3} \dfrac{x - 3}{4 - \sqrt{19 - x}}$

$\quad = \underset{x \to 3}{\text{had}} \dfrac{(x - 3)(4 + \sqrt{19 - x}}{16 - (19 - x)} / \lim_{x \to 3} \dfrac{(x - 3)(4 + \sqrt{19 - x}}{16 - (19 - x)}$

$\quad = \underset{x \to 3}{\text{had}} \dfrac{(x - 3)(4 + \sqrt{19 - x}}{x - 3} / \lim_{x \to 3} \dfrac{(x - 3)(4 + \sqrt{19 - x}}{x - 3}$

$\quad = \underset{x \to 3}{\text{had}}\, 4 + \sqrt{19 - x} / \lim_{x \to 3} 4 + \sqrt{19 - x}$

$\quad = 4 + \sqrt{19 - 3}$

$\quad = 8$

(b) $y = 5x^0$

$\dfrac{dy}{dx} = 0(5x^{-1})$

$\therefore k = 0, m = -1$

5 $\dfrac{dy}{dx} = 4x + 7$

$4x + 7 = 5$

$x = -\dfrac{1}{2}$

$y = 2\left(-\dfrac{1}{2}\right)^2 + 7\left(-\dfrac{1}{2}\right) - 1$

$\quad = -4$

$\therefore$ Koordinat/*Coordinates* $= \left(-\dfrac{1}{2}, -4\right)$

$y = 5x + p$ merupakan tangen kepada lengkung pada titik $\left(-\dfrac{1}{2}, -4\right)$ kerana $m_t = 5$.

$y = 5x + p$ is the tangent to the curve at point $\left(-\dfrac{1}{2}, -4\right)$ because $m_t = 5$.

$\therefore \quad y - (-4) = 5\left[x - \left(-\dfrac{1}{2}\right)\right]$

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$$y = 5x + \frac{5}{2} - 4$$
$$= 5x - \frac{3}{2}$$
$$\therefore \quad p = -\frac{3}{2}$$

6 $m_t = \frac{dy}{dx}$
$= 9x^2 - 36x + 42$
$= 9(x^2 - 4x) + 42$
$= 9[(x - 2)^2 - 4] + 42$
$= 9(x - 2)^2 - 36 + 42$
$= 9(x - 2)^2 + 6$
$\therefore m_t \geqslant 6$
$\therefore$ m_t tidak bernilai negatif untuk semua x.
m_t is never negative for all the values of x.

Kertas 2

1 $\frac{dy}{dx} = 6x^2 + 1$
$x = 2, \frac{dy}{dx} = 6(2)^2 + 1$
$= 25$
$\delta x = 0.02p$
$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$
$\delta y \approx \frac{dy}{dx} \times \delta x$
$= 25 \times 0.02p$
$\delta y = 0.5p$

2 $y_1 = y_2$
$x^2 - x - 5 = x^2 - \frac{31}{5}x + \frac{53}{5}$
$5x^2 - 5x - 25 = 5x^2 - 31x + 53$
$26x = 78$
$x = 3$
$y = (3)^2 - (3) - 5$
$= 1$
$\therefore \quad A(3, 1)$

(a) $m_{t_1} = \frac{dy_1}{dx}$
$= 2x - 1$
$m_{t_2} = \frac{dy_2}{dx}$
$= 2x - \frac{31}{5}$

(b) Pada/*At* $x = 3$,
$m_{t_1} = 2(3) - 1$
$= 5$
$m_{t_2} = 2(3) - \frac{31}{5}$
$= -\frac{1}{5}$
$m_{t_1} \times m_{t_2} = 5 \times -\frac{1}{5}$
$= -1$
(Tertunjuk bahawa tangen kedua-dua lengkung itu ialah normal antara satu sama lain.)
(*Shown that the tangents of both curves are normal to each other.*)

3 $m_t = \frac{dy}{dx}$
$= 2x + 2$
$x = 2, m_t = 2(2) + 2$
$= 6$
$\therefore \ m_n = -\frac{1}{6}$

Persamaan normal/*Equation of normal:*
$y = -\frac{1}{6}x + c$
$3 = -\frac{1}{6}(2) + c$
$c = \frac{10}{3}$
$y = -\frac{1}{6}x + \frac{10}{3}$
$\therefore \quad a = -\frac{1}{6}, b = \frac{10}{3}$

4 Biar/*Let* $y = x^{-\frac{1}{2}}$
$\frac{dy}{dx} = -\frac{1}{2}x^{-\frac{3}{2}}$
$x = 9, y = (9)^{-\frac{1}{2}}$
$= \frac{1}{3}$
$\frac{dy}{dx} = -\frac{1}{2}(9)^{-\frac{3}{2}}$
$= -\frac{1}{54}$
$\delta x = 0.02, \ \frac{\delta y}{\delta x} \approx \frac{dy}{dx}$
$\delta y \approx \frac{dy}{dx} \times \delta x$
$= -\frac{1}{54} \times 0.02$
$\delta y = -\frac{1}{2\ 700}$
$\therefore \quad y_n = y_o + \delta y$
$(9.02)^{-\frac{1}{2}} = \frac{1}{3} - \frac{1}{2\ 700}$
$= \frac{899}{2\ 700}$

5 (a) $V = (60 - x)(120 - 2x)(x)$
$= x(7\ 200 - 120x - 120x + 2x^2)$
$= 2x^3 - 240x^2 + 7\ 200x$ (tertunjuk *shown*)

(b) V mempunyai nilai pegun apabila
V has a stationary value when
$\frac{dV}{dx} = 0$
$6x^2 - 480x + 7\ 200 = 0$
$x^2 - 80x + 1\ 200 = 0$
$(x - 20)(x - 60) = 0$
$x = 20$($V \neq 0$, $\therefore$ 60 ditolak/*rejected*)
$= 12x - 480$
$x = 20, \frac{dV^2}{dx^2} = 12(20) - 480$
$= -240 < 0$

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$\therefore$ V maksimum/*maximum*
$= 2(20)^3 - 240(20)^2 + 7\,200(20)$
$= 64\,000 \text{ cm}^3$

6 (a) $\theta = \dfrac{s}{r}$

Dalam sebutan r/*In terms of r,*

$\theta = \dfrac{26 - 2r}{r}$

(b) $\dfrac{dr}{dt} = 0.1 \text{ cm s}^{-1}$, $r = 2$ cm,

(i) $\dfrac{d\theta}{dt} = \dfrac{d\theta}{dr} \times \dfrac{dr}{dt}$

$= \dfrac{r(-2) - (26 - 2r)(1)}{r^2} \times 0.1$

$= \dfrac{-2r - 26 + 2r}{r^2} \times 0.1$

$= \dfrac{-26}{(2)^2} \times 0.1$

$= -\,0.65 \text{ rad s}^{-1}$

(ii) $A = \dfrac{1}{2}r^2\left(\dfrac{26 - 2r}{r}\right)$

$= \dfrac{26 - 2r^2}{2}$

$\dfrac{dr}{dt} = 13 - 2r$

$\dfrac{dA}{dt} = \dfrac{dA}{dr} \times \dfrac{dr}{dt}$

$= [13 - 2(2)] \times 0.1$

$= 0.9 \text{ cm}^2\text{s}^{-1}$

7 (a) $\dfrac{x}{h} = \dfrac{24}{20}$

$x = \dfrac{6}{5}h$

$V = \dfrac{1}{2}(x + 16)\,(h)(30)$

$= \left(\dfrac{6}{5}h + 16\right)(h)(15)$

$= 18h^2 + 240h$ (tertunjuk/*shown*)

(b) $\dfrac{dV}{dt} = 60 \text{ cm}^3 \text{ s}^{-1}$, $h = 10$

$\dfrac{dV}{dh} = 36h + 240$

$\dfrac{dh}{dV} = \dfrac{dh}{dt} \times \dfrac{dV}{dt}$

$= \dfrac{1}{36(10) + 240} \times 60$

$= 0.1 \text{ cms}^{-1}$

8 $V = \dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3}\pi(12^2 - h^2)h$

$= \dfrac{1}{3}\pi(144 - h^2)h$

$= \dfrac{\pi}{3}(144h - h^3)$

V maksimum apabila/*V is maximum when* $\dfrac{dV}{dh} = 0$,

$\dfrac{\pi}{3}(144 - 3h^2) = 0$

$3h^2 = 144$

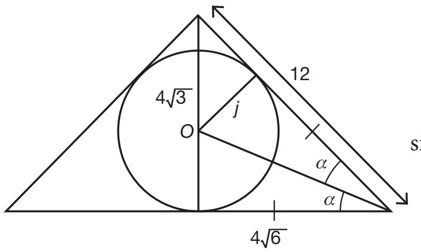
$h = \sqrt{48}$

$= 4\sqrt{3}$

$r^2 = 144 - h^2$

$= 144 - 48$

$r = 4\sqrt{6}$



$\sin 2\alpha = \dfrac{4\sqrt{3}}{12}$

$2\alpha = \sin^{-1}\dfrac{\sqrt{3}}{3}$

$= 35.26°$

$\alpha = 17.63°$

$\tan 17.63° = \dfrac{j}{4\sqrt{6}}$

$j = 3.114$ cm

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