

Tingkatan 4 Bab 4
Indeks, Surd dan Logaritma
Penyelesaian Lengkap

Praktis Formatif 4.1a

$$\begin{aligned} 1 \text{ (a)} \quad & 2^{3d+1} \times 2^{-3d} \times 8 \\ & = 2^{3d+1-3d+3} \\ & = 2^4 \\ & = 16 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & 3^{1-n} \times 27^{n+1} \div 9^{n+1} \\ & = 3^{1-n+3n+3-2n-2} \\ & = 3^2 \\ & = 9 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad & 5^3 \times 25^{2n-1} \div 5^{4n+1} \\ & = 5^{3+4n-2-4n-1} \\ & = 5^0 \\ & = 1 \end{aligned}$$

$$\begin{aligned} 2 \text{ (a)} \quad & \frac{2x^{-\frac{1}{3}}y^{-\frac{2}{3}}}{(x^2y^4)^{-\frac{1}{6}}} \\ & = \frac{2x^{-\frac{1}{3}}y^{-\frac{2}{3}}}{x^{-\frac{1}{3}}y^{-\frac{2}{3}}} \\ & = 2 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & \frac{3^{3n} \times 3^{2-n}}{3^4 \times 3^{2n+1}} \\ & = 3^{3n+2-n-4-2n-1} \\ & = 3^{-3} \\ & = \frac{1}{27} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad & \left(\frac{2x^2}{3y}\right)^3 \left(\frac{9y^2}{4x^3}\right)^2 \\ & = \frac{8x^6}{27y^3} \times \frac{81y^4}{16x^6} \\ & = \frac{3}{2}y \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad & \frac{25^m \times 10^{m+1}}{2^{m-1} \times 5^{3m+2}} \\ & = \frac{25^m \times (2 \times 5)^{m+1}}{2^{m-1} \times 5^{3m+2}} \\ & = \frac{25^m \times 2^{m+1} \times 5^{m+1}}{2^{m-1} \times 5^{3m+2}} \end{aligned}$$

$$\begin{aligned} & = 5^{2m+m+1-3m-2} 2^{m+1-m+1} \\ & = 5^{-1} \times 2^2 \\ & = \frac{4}{5} \end{aligned}$$

$$\begin{aligned} 3 \quad & 2^{n+4} - 2^{n+2} - 2^{2n} \\ & = 2^n 2^4 - 2^n 2^2 - 2^n \\ & = (16 - 4 - 1)2^n \\ & = 11(2^n) \\ & k = 11 \end{aligned}$$

$$\begin{aligned} 4 \quad & 3^{n+1} + 3^n + 9(3^{n-2}) \\ & = 3^n 3 + 3^n + 9\left(\frac{3^n}{9}\right) \\ & = (3+1+1)(3^n) \\ & = 5(3^n) \\ & h = 5 \end{aligned}$$

$$\begin{aligned} 5 \quad & 5^{n+2} - 7(5^n) - 25(5^{n-1}) \\ & = 5^n 5^2 - 7(5^n) - 25\left(\frac{5^n}{5}\right) \\ & = (25 - 7 - 5)(5^n) \\ & = 13(5^n) \\ & q = 13 \end{aligned}$$

Praktis Formatif 4.1b

$$\begin{aligned} 1 \text{ (a)} \quad & 8(4^x) = \frac{32}{16^{1-2x}} \\ & 2^{3+2x} = 2^{5-4(1-2x)} \\ & 3+2x = 5-4(1-2x) \\ & 3+2x = 5-4+8x \\ & 6x = 2 \\ & x = \frac{1}{3} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & \frac{3^y}{81^{y-1}} = \frac{1}{9^y} \\ & \frac{3^y}{3^{4(y-1)}} = 3^{-2y} \\ & 3^{y-4(y-1)} = 3^{-2y} \\ & y-4(y-1) = -2y \\ & y-4y+4 = -2y \\ & -y = -4 \\ & y = 4 \end{aligned}$$

$$(c) \frac{25^{3t-1}}{5^{2-t}} = 125^{t+3}$$

$$\frac{5^{2(3t-1)}}{5^{(2-t)}} = 5^{3(t+3)}$$

$$5^{6t-2-(2-t)} = 5^{3t+9}$$

$$6t-2-2+t = 3t+9$$

$$4t = 13$$

$$t = \frac{13}{4}$$

$$2 \frac{3}{x^{-\frac{4}{5}}} = 12x^{\frac{2}{5}}$$

$$3 = 12x^{\frac{2}{5}} \times x^{-\frac{4}{5}}$$

$$3 = 12x^{\frac{2}{5} - \frac{4}{5}}$$

$$\frac{1}{4} = x^{-\frac{2}{5}}$$

$$2^{-2} = x^{-\frac{2}{5}}$$

$$x = 2^{-2 \times -\frac{5}{2}}$$

$$x = 2^5 = 32$$

$$3(a) \frac{125^m}{25^n} = 625$$

$$\frac{5^{3m}}{5^{2n}} = 5^4$$

$$5^{3m-2n} = 5^4$$

$$3m-2n = 4 \dots (1)$$

$$2 \times 4^m = 32^n$$

$$2^{1+2m} = 2^{5n}$$

$$1+2m = 5n$$

$$2m-5n = -1 \dots (2)$$

$$\begin{array}{r} 6m-4n=8 \dots (1) \times 2 \\ (-) 6m-15n=-3 \dots (2) \times 3 \\ \hline 11n=11 \\ n=1 \end{array}$$

Daripada (1) :

$$3m-2(1) = 4$$

$$m = 2$$

$$(b) 3^p \times 9^{2q} = 27$$

$$3^{p+4q} = 3^3$$

$$p+4q = 3 \dots (1)$$

$$\frac{5^p}{25^q} = \frac{1}{125}$$

$$5^{p-2q} = 5^{-3}$$

$$p-2q = -3 \dots (2)$$

$$(1) - (2):$$

$$6q = 6$$

$$q = 1$$

Daripada (1) :

$$p+4(1) = 3$$

$$p = -1$$

$$4(a) 5^{n+2} - 5^{n+1} - 5^n = 19$$

$$5^n 5^2 - 5^n 5 - 5^n = 19$$

$$25(5^n) - 5(5^n) - 5^n = 19$$

$$19(5^n) = 19$$

$$5^n = 1$$

$$n = 0$$

$$(b) 2(3^{r+2}) - 3^{r+1} = 45$$

$$2(3^r)(3^2) - 3^r(3) = 45$$

$$(18-3)(3^r) = 45$$

$$15(3^r) = 45$$

$$3^r = 3$$

$$r = 1$$

$$5 5^{m+1} = 7^{n-1} = 35^k = p$$

$$5 = p^{\frac{1}{m+1}} \quad 7 = p^{\frac{1}{n-1}} \quad 35 = p^{\frac{1}{k}}$$

$$5 \times 7 = 35$$

$$\frac{1}{p^{m+1}} \times \frac{1}{p^{n-1}} = \frac{1}{p^k}$$

$$\frac{1}{m+1} + \frac{1}{n-1} = \frac{1}{k}$$

$$\frac{(n-1) + (m+1)}{(m+1)(n-1)} = \frac{1}{k}$$

$$\frac{m+n}{(m+1)(n-1)} = \frac{1}{k}$$

$$k(m+n) = (m+1)(n-1) \text{ [Tertunjuk]}$$

$$6 \quad 3^h = 11^k = 33^m = p$$

$$3 = p^{\frac{1}{h}} \quad 11 = p^{\frac{1}{k}} \quad 33 = p^{\frac{1}{m}}$$

$$3 \times 11 = 33$$

$$p^{\frac{1}{h}} \times p^{\frac{1}{k}} = p^{\frac{1}{m}}$$

$$\frac{1}{h} + \frac{1}{k} = \frac{1}{m}$$

$$\frac{h+k}{hk} = \frac{1}{m}$$

$$m = \frac{hk}{h+k}$$

$$7 \quad 120\,000(0.95)^n = 102\,885$$

$$0.95^n = 0.857375$$

$$0.95^n = 0.95^3$$

$$n = 3 \text{ tahun}$$

Praktis Formatif 4.2a

$$1 \text{ (a)} \quad \sqrt{50} = \sqrt{25(2)} = 5\sqrt{2}$$

$$\text{(b)} \quad \sqrt{147} = \sqrt{49 \times 3} = 7\sqrt{3}$$

$$\text{(c)} \quad \sqrt{288} = \sqrt{144(2)} = 12\sqrt{2}$$

$$\text{(d)} \quad \sqrt{450} = \sqrt{225 \times 2} = 15\sqrt{2}$$

$$\text{(e)} \quad \sqrt{800} = \sqrt{400 \times 2} = 20\sqrt{2}$$

$$2 \text{ (a)} \quad \sqrt{3} \times \sqrt{27} = \sqrt{81} = 9$$

$$\text{(b)} \quad 6\sqrt{6} \times 3\sqrt{24} = 18\sqrt{144} = 18(12) = 216$$

$$\text{(c)} \quad \sqrt{18} \times \sqrt{125} \div \sqrt{2}$$

$$= \frac{\sqrt{9 \times 2} \times \sqrt{25 \times 5}}{\sqrt{2}}$$

$$= \frac{3\sqrt{2} \times 5\sqrt{5}}{\sqrt{2}}$$

$$= 15\sqrt{5}$$

$$3 \text{ (a)} \quad \sqrt{12} - 3\sqrt{27} + \sqrt{48}$$

$$= 2\sqrt{3} - 3 \times 3\sqrt{3} + 4\sqrt{3}$$

$$= -3\sqrt{3}$$

$$\text{(b)} \quad \sqrt{45} + \sqrt{125} - 2\sqrt{5}$$

$$= 3\sqrt{5} + 5\sqrt{5} - 2\sqrt{5}$$

$$= 6\sqrt{5}$$

$$\text{(c)} \quad \sqrt{27} - \sqrt{12} + 2\sqrt{75}$$

$$= 3\sqrt{3} - 2\sqrt{3} + 2 \times 5\sqrt{3}$$

$$= 11\sqrt{3}$$

$$\text{(d)} \quad \frac{3\sqrt{50}}{5} \times \frac{5\sqrt{48}}{4} \div \sqrt{24}$$

$$= \frac{\frac{3}{5} \times 5\sqrt{2} \times \frac{5}{4} \times 4\sqrt{3}}{2\sqrt{6}}$$

$$= \frac{3\sqrt{2} \times 3\sqrt{3}}{2\sqrt{6}}$$

$$= \frac{15}{2}$$

$$4 \text{ (a)} \quad \frac{5}{\sqrt{3}} = 5 \frac{\sqrt{3}}{3}$$

$$\text{(b)} \quad \frac{7}{2\sqrt{2}} = \frac{7}{2} \times \frac{\sqrt{2}}{2} = \frac{7}{4}\sqrt{2}$$

$$\text{(c)} \quad \sqrt{\frac{3}{5}} = \frac{\sqrt{3}}{\sqrt{5}} = \frac{\sqrt{3} \times \sqrt{5}}{5} = \frac{\sqrt{15}}{5}$$

$$5 \text{ (a)} \quad \frac{1}{3-\sqrt{7}} + \frac{1}{3+\sqrt{7}}$$

$$= \frac{3+\sqrt{7}+3-\sqrt{7}}{(3-\sqrt{7})(3+\sqrt{7})}$$

$$= \frac{6}{9-7}$$

$$= 3$$

$$\text{(b)} \quad \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$= \frac{(\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$$

$$= \frac{3-\sqrt{6}-\sqrt{6}+2}{3-2}$$

$$= 5-2\sqrt{6}$$

$$\begin{aligned}
 \text{(c)} \quad & \frac{3\sqrt{5}-\sqrt{2}}{2\sqrt{5}+3\sqrt{2}} \\
 &= \frac{(3\sqrt{5}-\sqrt{2})(2\sqrt{5}-3\sqrt{2})}{(2\sqrt{5}+3\sqrt{2})(2\sqrt{5}-3\sqrt{2})} \\
 &= \frac{6(5)-9\sqrt{10}-2\sqrt{10}+3(2)}{20-18} \\
 &= \frac{36-11\sqrt{10}}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad & \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} \\
 &= \frac{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}-2\sqrt{3})}{(3\sqrt{2}+2\sqrt{3})(3\sqrt{2}-2\sqrt{3})} \\
 &= \frac{9(2)-6\sqrt{6}-6\sqrt{6}+4(3)}{18-12} \\
 &= \frac{30-12\sqrt{6}}{6} \\
 &= \frac{5-2\sqrt{6}}{1} \\
 &= 5-2\sqrt{6}
 \end{aligned}$$

Praktis Formatif 4.2b

$$\begin{aligned}
 \mathbf{1} \quad & \sqrt{[2-(-4)]^2 + [-5-(-3)]^2} \\
 &= \sqrt{36+4} \\
 &= \sqrt{40} \\
 &= 2\sqrt{10} \text{ unit}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{2} \quad & \tan 60^\circ \times \sin 45^\circ \\
 &= \sqrt{3} \times \frac{1}{\sqrt{2}} \\
 &= \sqrt{3} \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\
 &= \sqrt{3} \times \frac{\sqrt{2}}{2} \\
 &= \frac{\sqrt{6}}{2}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{3} \quad & \frac{x^2}{45} + \frac{y^2}{50} = 1 \\
 x\text{-intercept} &= \pm\sqrt{45} = \pm 3\sqrt{5} \\
 y\text{-intercept} &= \pm\sqrt{50} = \pm 5\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{4} \text{ (a)} \quad & \sqrt{2x-1} - \sqrt{x+3} = 1 \\
 & (\sqrt{2x-1} - \sqrt{x+3})^2 = 1^2 \\
 2x-1-2\sqrt{2x-1}\sqrt{x+3}+x+3 &= 1 \\
 3x+1 &= 2\sqrt{2x-1}\sqrt{x+3} \\
 (3x+1)^2 &= (2\sqrt{2x-1}\sqrt{x+3})^2 \\
 9x^2+6x+1 &= 4(2x-1)(x+3) \\
 9x^2+6x+1 &= 4(2x^2+5x-3) \\
 9x^2+6x+1 &= 8x^2+20x-12 \\
 x^2-14x+13 &= 0 \\
 (x-1)(x-13) &= 0 \\
 x=1 \text{ atau } x=13 \\
 x=1 & \text{ tidak diterima kerana ia tidak} \\
 & \text{memuaskan persamaan asal.} \\
 \therefore x &= 13
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & \sqrt{3x+4} - \sqrt{x+2} = \sqrt{x-3} \\
 & (\sqrt{3x+4} - \sqrt{x+2})^2 = (\sqrt{x-3})^2 \\
 3x+4-2\sqrt{3x+4}\sqrt{x+2}+x+2 &= x-3 \\
 3x+9 &= 2\sqrt{3x+4}\sqrt{x+2} \\
 (3x+9)^2 &= (2\sqrt{3x+4}\sqrt{x+2})^2 \\
 9x^2+54x+81 &= 4(3x+4)(x+2) \\
 9x^2+54x+81 &= 4(3x^2+10x+8) \\
 9x^2+54x+81 &= 12x^2+40x+32 \\
 3x^2-14x-49 &= 0 \\
 (x-7)(3x+7) &= 0 \\
 x=7 \text{ atau } x &= -\frac{7}{3} \\
 x &= -\frac{7}{3} \text{ tidak diterima kerana ia tidak} \\
 & \text{memuaskan persamaan asal.} \\
 \therefore x &= 7
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad & \sqrt{2p-1} + \sqrt{p-1} = 5 \\
 & (\sqrt{2p-1} + \sqrt{p-1})^2 = 5^2 \\
 2p-1+2\sqrt{2p-1}\sqrt{p-1}+p-1 &= 25 \\
 2\sqrt{2p-1}\sqrt{p-1} &= 27-3p \\
 (2\sqrt{2p-1}\sqrt{p-1})^2 &= (27-3p)^2 \\
 4(2p-1)(p-1) &= 729-162p+9p^2 \\
 4(2p^2-3p+1) &= 729-162p+9p^2 \\
 8p^2-12p+4 &= 729-162p+9p^2 \\
 p^2-150p+725 &= 0 \\
 (p-5)(p-145) &= 0
 \end{aligned}$$

$p = 5$ atau $p = 145$
 $p = 145$ tidak diterima kerana ia tidak
 memuaskan persamaan asal.
 $\therefore p = 5$

Praktis Formatif 4.3a

1 (a) $11^2 = 121$
 $2 = \log_{11} 121$

(b) $7^0 = 1$
 $0 = \log_7 1$

(c) $5^{-3} = \frac{1}{125}$
 $-3 = \log_5 \frac{1}{125}$

2 (a) $\log_2 x = 3$
 $x = 2^3 = 8$

(b) $\log_3 x = -4$
 $x = 3^{-4} = \frac{1}{81}$

(c) $\log_x 64 = 3$
 $64 = x^3$
 $x = 4$

(d) $\log_x 5 = \frac{1}{2}$
 $5 = x^{\frac{1}{2}}$
 $x = 5^2 = 25$

3 (a) $\log_{10} 0.1945$
 $= -0.7111$

(b) $\log_{10} 0.7261$
 $= -0.1390$

(c) $\log_{10} 7.314$
 $= 0.8642$

(d) $\lg 335.7$
 $= 2.5260$

4 (a) $\log_{10} x = 0.9566$
 $x = 9.049$

(b) $\log_{10} x = 0.7443$

$x = 5.550$
 (c) $\lg x = 1.8151$
 $x = 65.33$

(d) $\lg x = 2.0986$
 $x = 125.49$

5 (a) $\log_3 81$
 $= \log_3 3^4$
 $= 4$

(b) $\log_5 125$
 $= \log_5 5^3$
 $= 3$

(c) $\log_{64} 8$
 $= \log_{64} 64^{\frac{1}{2}}$
 $= \frac{1}{2}$

6 (a) $\log_a x^3 y z^5$
 $= \log_a x^3 + \log_a y + \log_a z^5$
 $= 3 \log_a x + \log_a y + 5 \log_a z$
 $= 3p + q + 5r$

(b) $\log_a \frac{x^2 \sqrt{y}}{z^3}$
 $= \log_a x^2 + \log_a y^{\frac{1}{2}} - \log_a z^3$
 $= 2 \log_a x + \frac{1}{2} \log_a y - 3 \log_a z$
 $= 2p + \frac{1}{2}q - 3r$

(c) $\log_a \frac{x^4}{y^3 z^2}$
 $= \log_a x^4 - (\log_a y^3 + \log_a z^2)$
 $= 4 \log_a x - (3 \log_a y + 2 \log_a z)$
 $= 4p - 3q - 2r$

7 (a) $\log_2 \left(\frac{25}{9} \right)$
 $= \log_2 \left(\frac{5^2}{3^2} \right)$
 $= \log_2 5^2 - \log_2 3^2$
 $= 2k - 2m$

(b) $\log_2 360$

$$\begin{aligned} &= \log_2 (2^3 \times 3^2 \times 5) \\ &= \log_2 2^3 + \log_2 3^2 + \log_2 5 \\ &= 3 \log_2 2 + 2 \log_2 3 + \log_2 5 \\ &= 3 + 2m + k \end{aligned}$$

2	360
2	180
2	90
3	45
3	15
5	5
	1

(c) $\log_2 \sqrt{60}$

$$\begin{aligned} &= \log_2 (2 \times 2 \times 3 \times 5)^{\frac{1}{2}} \\ &= \frac{1}{2} \log_2 (2 \times 2 \times 3 \times 5) \\ &= \frac{1}{2} (2 + m + k) \\ &= 1 + \frac{1}{2}m + \frac{1}{2}k \end{aligned}$$

(d) $\log_2 7.5$

$$\begin{aligned} &= \log_2 \left(\frac{15}{2} \right) \\ &= \log_2 \left(\frac{3 \times 5}{2} \right) \\ &= \log_2 3 + \log_2 5 - \log_2 2 \\ &= m + k - 1 \end{aligned}$$

8 (a) $\log_a 200$

$$\begin{aligned} &= \log_a (2^3 \times 5^2) \\ &= 3 \log_a 2 + 2 \log_a 5 \\ &= 3h + 2k \end{aligned}$$

2	200
2	100
2	50
5	25
5	5
	1

(b) $\log_a \left(\frac{4}{25} \right)$

$$\begin{aligned} &= \log_a \left(\frac{2^2}{5^2} \right) \\ &= 2 \log_a 2 - 2 \log_a 5 \\ &= 2h - 2k \end{aligned}$$

(c) $\log_a \frac{64\sqrt{a}}{125}$

$$\begin{aligned} &= \log_a \frac{2^6 a^{\frac{1}{2}}}{5^3} \\ &= 6 \log_a 2 + \frac{1}{2} \log_a a - 3 \log_a 5 \end{aligned}$$

$$= 6h + \frac{1}{2} - 3k$$

9 (a) $\log_5 360$

$$\begin{aligned} &= \log_5 (2^3 \times 3^2 \times 5) \\ &= 3 \log_5 2 + 2 \log_5 3 + \log_5 5 \\ &= 3m + 2n + 1 \end{aligned}$$

2	360
2	180
2	90
3	45
3	15
5	5
	1

(b) $\log_5 0.96$

$$\begin{aligned} &= \log_5 \left(\frac{24}{25} \right) \\ &= \log_5 \left(\frac{2^3 \times 3}{5^2} \right) \\ &= 3 \log_5 2 + \log_5 3 - 2 \log_5 5 \\ &= 3m + n - 2 \end{aligned}$$

10 (a) $2 \lg 4 + 2 \lg 5 - \lg 4$

$$\begin{aligned} &= \lg \left(\frac{4^2 \times 5^2}{4} \right) \\ &= \log_{10} (100) \\ &= 2 \end{aligned}$$

(b) $3 \log_2 5 - 6 \log_2 \left(\frac{1}{2} \right) - 3 \log_2 10$

$$\begin{aligned} &= \log_2 \left(\frac{5^3}{\frac{1}{2^6} \times 10^3} \right) \\ &= \log_2 \left(\frac{125 \times 64}{1000} \right) \\ &= \log_2 8 \\ &= \log_2 2^3 \\ &= 3 \end{aligned}$$

(c) $\log_2 \left[\frac{75}{16} \times \left(\frac{9}{5} \right)^2 \times \frac{32}{243} \right]$

$$\begin{aligned} &= \log_2 2 \\ &= 1 \end{aligned}$$

11 (a) $11^{\log_{11} 13} = 13$

(b) $7^{3 \log_7 5} = 7^{\log_7 5^3} = 125$

$$(c) 5^{\frac{1}{2} \log_5 49} = 5^{\log_5 \sqrt{49}} = 7$$

Praktis Formatif 4.3b

$$1 (a) \log_3 8 = \frac{\log_{10} 8}{\log_{10} 3} = 1.893$$

$$(b) \log_5 137 = \frac{\log_{10} 137}{\log_{10} 5} = 3.057$$

$$(c) \log_{0.3} 0.25 = \frac{\log_{10} 0.25}{\log_{10} 0.3} = 1.151$$

$$2 \log_3 125 \times \log_5 81$$

$$\begin{aligned} &= \frac{\log_{10} 125}{\log_{10} 3} \times \frac{\log_{10} 81}{\log_{10} 5} \\ &= \frac{\log_{10} 5^3}{\log_{10} 3} \times \frac{\log_{10} 3^4}{\log_{10} 5} \\ &= \frac{3 \log_{10} 5}{\log_{10} 3} \times \frac{4 \log_{10} 3}{\log_{10} 5} \\ &= 3 \times 4 \\ &= 12 \end{aligned}$$

$$3 (a) \log_2 30$$

$$\begin{aligned} &= \log_2 (2 \times 3 \times 5) \\ &= \log_2 2 + \log_2 3 + \log_2 5 \\ &= 1 + 1.585 + 2.322 \\ &= 4.907 \end{aligned}$$

$$(b) \log_2 \left(\frac{5}{6} \right)$$

$$\begin{aligned} &= \log_2 5 - \log_2 6 \\ &= \log_2 5 - \log_2 (2 \times 3) \\ &= \log_2 5 - \log_2 2 - \log_2 3 \\ &= 2.322 - 1 - 1.585 \\ &= -0.263 \end{aligned}$$

$$(c) \log_{15} 6$$

$$\begin{aligned} &= \frac{\log_2 6}{\log_2 15} \\ &= \frac{\log_2 (2 \times 3)}{\log_2 (3 \times 5)} \\ &= \frac{\log_2 2 + \log_2 3}{\log_2 3 + \log_2 5} \\ &= \frac{1 + 1.585}{1.585 + 2.322} \\ &= \frac{2.585}{3.907} \\ &= 0.662 \end{aligned}$$

$$4 (a) \log_5 42$$

$$\begin{aligned} &= \log_5 (2 \times 3 \times 7) \\ &= \log_5 2 + \log_5 3 + \log_5 7 \\ &= a + b + c \end{aligned}$$

$$(b) \log_5 2.1$$

$$\begin{aligned} &= \log_5 \left(\frac{21}{10} \right) \\ &= \log_5 \left(\frac{3 \times 7}{2 \times 5} \right) \\ &= \log_5 3 + \log_5 7 - \log_5 2 - \log_5 5 \\ &= b + c - a - 1 \end{aligned}$$

$$(c) \log_{25} \frac{6}{49} = \frac{\log_5 \frac{6}{49}}{\log_5 25}$$

$$\begin{aligned} &= \frac{\log_5 6 - \log_5 49}{\log_5 5^2} \\ &= \frac{\log_5 (2 \times 3) - \log_5 7^2}{\log_5 5^2} \\ &= \frac{\log_5 2 + \log_5 3 - 2 \log_5 7}{2} \\ &= \frac{a + b - 2c}{2} \end{aligned}$$

$$5 \quad \begin{array}{l|l} x = 3^r & y = 3^t \\ \log_3 x = r & \log_3 y = t \end{array}$$

$$(a) \log_3 \left(\frac{xy^3}{81} \right)$$

$$\begin{aligned} &= \log_3 x + 3 \log_3 y - \log_3 3^4 \\ &= r + 3t - 4 \end{aligned}$$

$$(b) \log_9 \left(\frac{x^3}{y^5} \right)$$

$$\begin{aligned} &= \frac{\log_3 x^3 - \log_3 y^5}{\log_3 9} \\ &= \frac{3 \log_3 x - 5 \log_3 y}{\log_3 3^2} \\ &= \frac{3r - 5t}{2} \end{aligned}$$

$$6 \quad \begin{array}{l|l} \log_{11} a = r & \log_7 a = t \\ \log_a 11 = \frac{1}{r} & \log_a 7 = \frac{1}{t} \end{array}$$

$$\begin{aligned} \log_a 77 &= \log_a (11 \times 7) \\ &= \log_a 11 + \log_a 7 \end{aligned}$$

$$= \frac{1}{r} + \frac{1}{t}$$

$$= \frac{t+r}{rt}$$

$$x = \frac{-2.9420}{0.9912}$$

$$x = -2.968$$

$$7 \log_{xy} a = \frac{\log_a a}{\log_a xy} = \frac{1}{\log_a xy}$$

$$= \frac{1}{\log_a x + \log_a y}$$

$$= \frac{1}{0.26 + 0.24}$$

$$= \frac{1}{0.50}$$

$$= 2$$

Praktis Formatif 4.3c

$$1 \text{ (a)} \quad 5^x = 7$$

$$x \lg 5 = \lg 7$$

$$x = \frac{\lg 7}{\lg 5}$$

$$x = 1.209$$

$$(b) \quad 5^{x+3} = 9$$

$$(x+3) \lg 5 = \lg 9$$

$$x \lg 5 + 3 \lg 5 = \lg 9$$

$$x = \frac{\lg 9 - 3 \lg 5}{\lg 5}$$

$$x = -1.635$$

$$(c) \quad 0.2^x = 0.7^{x+2}$$

$$x \lg 0.2 = (x+2) \lg 0.7$$

$$x \lg 0.2 = x \lg 0.7 + 2 \lg 0.7$$

$$x(\lg 0.2 - \lg 0.7) = 2 \lg 0.7$$

$$x = \frac{2 \lg 0.7}{\lg 0.2 - \lg 0.7}$$

$$x = \frac{2 \lg 0.7}{\lg 0.2 - \lg 0.7}$$

$$x = 0.5694$$

$$(d) \quad 7^{2x+1} = 5^{x-3}$$

$$(2x+1) \lg 7 = (x-3) \lg 5$$

$$2x \lg 7 - x \lg 5 = -\lg 7 - 3 \lg 5$$

$$x(2 \lg 7 - \lg 5) = -\lg 7 - 3 \lg 5$$

$$x = \frac{-\lg 7 - 3 \lg 5}{2 \lg 7 - \lg 5}$$

$$(e) \quad (2^x)(5^{x+2}) = 13$$

$$\lg 2^x + \lg 5^{x+2} = \lg 13$$

$$x \lg 2 + (x+2) \lg 5 = \lg 13$$

$$x(\lg 2 + \lg 5) = \lg 13 - 2 \lg 5$$

$$x = \frac{\lg 13 - 2 \lg 5}{\lg 2 + \lg 5}$$

$$x = -0.2840$$

$$2 \text{ (a)} \quad \log_7 6x = 2 \log_7 3$$

$$\log_7 6x = \log_7 3^2$$

$$6x = 9$$

$$x = \frac{3}{2}$$

$$(b) \quad \log_3 (x-2) = \frac{1}{2} \log_3 25$$

$$\log_3 (x-2) = \log_3 25^{\frac{1}{2}}$$

$$x-2 = 5$$

$$x = 7$$

$$(c) \quad \log_3 x - \log_3 5 = 4$$

$$\log_3 \left(\frac{x}{5} \right) = 4$$

$$\frac{x}{5} = 3^4$$

$$x = 405$$

$$(d) \quad \log_x 2 + \log_x 16 = 5$$

$$\log_x 32 = 5$$

$$32 = x^5$$

$$x = 2$$

$$(e) \quad 2 \log_x 3 + \frac{1}{2} \log_x 16 = 2$$

$$\log_x 3^2 + \log_x 16^{\frac{1}{2}} = 2$$

$$\log_x (9 \times 4) = 2$$

$$36 = x^2$$

$$x = 6$$

$$(f) \quad \log_x 10 + 2 \log_x 2 = 3 + 3 \log_x 3 + \log_x 5$$

$$\log_x \left(\frac{10 \times 4}{27 \times 5} \right) = 3$$

$$\frac{8}{27} = x^3$$

$$x = \frac{2}{3}$$

$$y = \frac{x^3}{3}$$

3 (a) $\log_2 (3x-1) = 1 + \log_2 (x+1)$

$$\log_2 (3x-1) - \log_2 (x+1) = 1$$

$$\log_2 \left(\frac{3x-1}{x+1} \right) = 1$$

$$\frac{3x-1}{x+1} = 2^1$$

$$3x-1 = 2x+2$$

$$x = 3$$

(b) $\log_2 5 + \log_2 (2x-1) = 1 + \log_2 (3x+1)$

$$\log_2 5 + \log_2 (2x-1) - \log_2 (3x+1) = 1$$

$$\log_2 \left[\frac{5(2x-1)}{3x+1} \right] = 1$$

$$\frac{10x-5}{3x+1} = 2^1$$

$$10x-5 = 6x+2$$

$$4x = 7$$

$$x = \frac{7}{4}$$

(c) $\log_3 (x+3) = 2 - \log_3 (x-5)$

$$\log_3 (x+3) + \log_3 (x-5) = 2$$

$$\log_3 (x+3)(x-5) = 2$$

$$x^2 - 2x - 15 = 3^2$$

$$x^2 - 2x - 24 = 0$$

$$(x+4)(x-6) = 0$$

$$x = 4 \text{ atau } x = 6$$

$$x = 4 \text{ tidak diterima.}$$

$$\therefore x = 6$$

4 (a) $\log_9 y + 4 \log_9 x = 2$

$$\log_9 y + \log_9 x^4 = 2$$

$$\log_9 x^4 y = 2$$

$$x^4 y = 81$$

$$y = \frac{81}{x^4}$$

(b) $\log_3 y + 1 = 3 \log_3 x$

$$\log_3 y - \log_3 x^3 = -1$$

$$\log_3 \frac{y}{x^3} = -1$$

$$\frac{y}{x^3} = 3^{-1}$$

5 (a) $\log_{25} (x-1) = \log_5 (x-3)$

$$\frac{\log_5 (x-1)}{\log_5 25} = \log_5 (x-3)$$

$$\frac{\log_5 (x-1)}{\log_5 5^2} = \log_5 (x-3)$$

$$\log_5 (x-1) = 2 \log_5 (x-3)$$

$$\log_5 (x-1) = \log_5 (x-3)^2$$

$$x-1 = (x-3)^2$$

$$x-1 = x^2 - 6x + 9$$

$$x^2 - 7x + 10 = 0$$

$$(x-2)(x-5) = 10$$

$$x = 2 \text{ tidak diterima.}$$

$$\therefore x = 5$$

(b) $2 + 2 \log_4 (x-6) = \log_2 x$

$$2 + \frac{2 \log_2 (x-6)}{\log_2 4} = \log_2 x$$

$$2 + \frac{2 \log_2 (x-6)}{\log_2 2^2} = \log_2 x$$

$$2 + \log_2 (x-6) = \log_2 x$$

$$\log_2 (x-6) - \log_2 x = -2$$

$$\log_2 \frac{x-6}{x} = -2$$

$$\frac{x-6}{x} = \frac{1}{4}$$

$$4x - 24 = x$$

$$3x = 24$$

$$x = 8$$

6 (a) $\log_{32} y = \log_2 x$

$$\frac{\log_2 y}{\log_2 32} = \log_2 x$$

$$\frac{\log_2 y}{5} = \log_2 x$$

$$\log_2 y = 5 \log_2 x$$

$$\log_2 y = \log_2 x^5$$

$$y = x^5$$

(b) $\log_9 y - \log_3 x = \frac{1}{2}$

$$\frac{\log_3 y}{\log_3 9} - \log_3 x = \frac{1}{2}$$

$$\frac{\log_3 y}{2} - \log_3 x = \frac{1}{2}$$

$$\log_3 y - 2\log_3 x = 1$$

$$\log_3 \frac{y}{x^2} = 1$$

$$\frac{y}{x^2} = 3$$

$$y = 3x^2$$

7 $400\,000(1.02)^t > 500\,000$

$$1.02^t > \frac{5}{4}$$

$$t \lg 1.02 > \lg \left(\frac{5}{4}\right)$$

$$t > \frac{\lg 1.25}{\lg 1.02}$$

$$t > 11.27$$

$$t_{\min} = 12 \text{ tahun}$$

Praktis Formatif 4.4

1 $450\,000(1.03)^t > 600\,000$

$$1.03^t > \frac{60}{45}$$

$$t \lg 1.03 > \lg \left(\frac{4}{3}\right)$$

$$t > \frac{\lg \left(\frac{4}{3}\right)}{\lg 1.03}$$

$$t > 9.73$$

$$t_{\text{minimum}} = 10 \text{ tahun}$$

2 $2\,000(1.05)^t > 4\,000$

$$1.05^t > 2$$

$$t \lg 1.05 > \lg 2$$

$$t > \frac{\lg 2}{\lg 1.05}$$

$$t > 14.21$$

$$t_{\text{minimum}} = 15 \text{ tahun}$$

3 $5000(0.92)^n < 3000$

$$0.92^n < \frac{3}{5}$$

$$n \lg 0.92 < \lg 0.6$$

$$n > \frac{\lg 0.6}{\lg 0.92}$$

$$n > 6.13$$

$$n_{\text{minimum}} = 7 \text{ tahun}$$

4 $j_0 e^{-0.2t} = \frac{j_0}{2}$

$$e^{-0.2t} = \frac{1}{2}$$

$$-0.2t = \log_e (0.5)$$

$$-0.2t = \ln 0.5$$

$$-0.2t = -0.6931$$

$$t = 3.466 \text{ tahun}$$

5 $\sqrt{3x+4} - \sqrt{x+2} = \sqrt{x-3}$

$$\left(\sqrt{3x+4} - \sqrt{x+2}\right)^2 = \left(\sqrt{x-3}\right)^2$$

$$3x+4 - 2\sqrt{3x+4}\sqrt{x+2} + x+2 = x-3$$

$$3x+9 = 2\sqrt{3x+4}\sqrt{x+2}$$

$$(3x+9)^2 = 4(3x+4)(x+2)$$

$$9x^2 + 54x + 81 = 4(3x^2 + 10x + 8)$$

$$3x^2 - 14x - 49 = 0$$

$$(x-7)(3x+7) = 0$$

$$x = 7 \text{ atau } x = -\frac{7}{3}$$

$$x = -\frac{7}{3} \text{ tidak diterima.}$$

$$\therefore x = 7$$

6 $y = ax^n + 2$

Lengkung melalui titik (3, 7), maka $x = 3$ dan $y = 7$.

$$7 = a(3)^n + 2$$

$$5 = a(3)^n \dots (1)$$

Lengkung melalui titik (9, 52), maka $x = 9$ dan $y = 52$.

$$52 = a(9)^n + 2$$

$$50 = a(3^2)^n$$

$$50 = a(3^2)^n \dots (2)$$

$$\frac{(2)}{(1)} : \frac{a(3^2)^n}{a(3)^n} = \frac{50}{5}$$

$$3^n = 10$$

$$n \log_{10} 3 = \log_{10} 10$$

$$n = \frac{1}{\log_{10} 3}$$

$$n = 2.096$$

Gantikan $3^n = 10$ ke dalam (1).

$$5 = a(3)^n$$

$$5 = a(10)$$

$$a = \frac{1}{2}$$

Praktis Sumatif 4

$$\begin{aligned} 1 \quad & \frac{2^{3(2n-4)} \times 2^{2(n+1)}}{2^{-6n} \times 2^{14n-7}} \\ &= 2^{6n-12+2n+2+6n-14n+7} \\ &= 2^{-3} \\ &= \frac{1}{8} \end{aligned}$$

$$\begin{aligned} 2 \quad & \frac{(2 \times 3)^{\frac{n}{2}} \times (2^2 \times 3)^{n+1} \times 3^{-\frac{3n}{2}}}{2^{\frac{5n}{2}}} \\ &= 2^{\frac{n}{2}+2n+2-\frac{5n}{2}} \times 3^{\frac{n}{2}+n+1-\frac{3n}{2}} \\ &= 2^2 \times 3^1 \\ &= 12 \end{aligned}$$

$$\begin{aligned} 3 \quad & 3^m 3^3 - 3^m 3^2 - 3^3 \left(\frac{3^m}{3} \right) - 2(3^m) \\ &= (27 - 9 - 9 - 2)(3^m) \\ &= 7(3^m) \\ \therefore k &= 7 \end{aligned}$$

$$\begin{aligned} 4 \quad & 3^{y^2+3} = 9^{2y} \\ & 3^{y^2+3} = (3^2)^{2y} \\ & 3^{y^2+3} = 3^{4y} \\ \text{Menyamakan indeks :} \\ & y^2 + 3 = 4y \\ & y^2 - 4y + 3 = 0 \\ & (y-1)(y-3) = 0 \\ & y = 1 \text{ atau } 3 \end{aligned}$$

$$\begin{aligned} 5 \quad & 3^{x-1} = 36 - 3^x \\ & \frac{3^x}{3} = 36 - 3^x \\ & \frac{3^x}{3} + 3^x = 36 \\ & \frac{4}{3}(3^x) = 36 \\ & 3^x = \frac{36 \times 3}{4} = 27 \\ & x = 3 \end{aligned}$$

$$\begin{aligned} 6 \quad & 5^{x+2} + 125(5^{x-1}) = 1250 \\ & (5^x)(5^2) + 125 \left(\frac{5^x}{5} \right) = 1250 \\ & (25 + 25)(5^x) = 1250 \\ & 5^x = \frac{1250}{50} \\ & 5^x = 25 \\ & x = 2 \end{aligned}$$

$$\begin{aligned} 7 \quad & 27^{2x-5} = \frac{1}{\sqrt{9^{x+1}}} \\ & (3^3)^{2x-5} = [3^{2(x+1)}]^{-\frac{1}{2}} \\ \text{Menyamakan indeks :} \\ & 6x - 15 = -(x+1) \\ & 7x = 14 \\ & x = 2 \end{aligned}$$

$$\begin{aligned} 8 \quad & 2^a = 5^b = 20^c = k \\ & \begin{array}{l|l|l} 2^a = k & 5^b = k & 20^c = k \\ 2 = k^{\frac{1}{a}} & 5 = k^{\frac{1}{b}} & 20 = k^{\frac{1}{c}} \end{array} \\ & 2^2 \times 5 = 20 \\ & \frac{2}{k^a} \times \frac{1}{k^b} = \frac{1}{k^c} \\ & \frac{2}{k^a} \times \frac{1}{k^b} = \frac{1}{k^c} \\ & \frac{2}{a} + \frac{1}{b} = \frac{1}{c} \\ & \frac{2b+a}{ab} = \frac{1}{c} \\ & c = \frac{ab}{a+2b} \end{aligned}$$

$$\begin{aligned} 9(a) \quad & \frac{1}{3+\sqrt{2}} \\ &= \left(\frac{1}{3+\sqrt{2}} \right) \left(\frac{3-\sqrt{2}}{3-\sqrt{2}} \right) \\ &= \frac{3-\sqrt{2}}{9-2} \\ &= \frac{3-\sqrt{2}}{7} \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & \left(\frac{2\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}} \right) \left(\frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} - \sqrt{3}} \right) \\
 &= \frac{2(5) - 2\sqrt{15} - \sqrt{15} + 3}{5 - 3} \\
 &= \frac{13 - 3\sqrt{15}}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad & \left(\frac{3\sqrt{2} - 2\sqrt{3}}{3\sqrt{2} + 2\sqrt{3}} \right) \left(\frac{3\sqrt{2} - 2\sqrt{3}}{3\sqrt{2} - 2\sqrt{3}} \right) \\
 &= \frac{9(2) - 6\sqrt{6} - 6\sqrt{6} + 4(3)}{9(2) - 4(3)} \\
 &= \frac{30 - 12\sqrt{6}}{6} \\
 &= 5 - 2\sqrt{6}
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad & \frac{1}{(1 + \sqrt{2})^2} + \frac{1}{(1 - \sqrt{2})^2} \\
 &= \frac{1}{1 + 2 + 2\sqrt{2}} + \frac{1}{1 + 2 - 2\sqrt{2}} \\
 &= \frac{1}{3 + 2\sqrt{2}} + \frac{1}{3 - 2\sqrt{2}} \\
 &= \frac{3 - 2\sqrt{2} + 3 + 2\sqrt{2}}{9 - 4(2)} \\
 &= \frac{6}{1} \\
 &= 6
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{10} \quad \log_2 70 &= \log_2 (2 \times 5 \times 7) \\
 &= \log_2 2 + \log_2 5 + \log_2 7 \\
 &= 1 + p + q
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{11} \quad \log_3 \left(2 \frac{1}{7} \right) &= \log_3 \left(\frac{15}{7} \right) \\
 &= \log_3 \left(\frac{3 \times 5}{7} \right) \\
 &= \log_3 3 + \log_3 5 - \log_3 7 \\
 &= 1 + m - q
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{12} \quad & \begin{array}{l} m = 2^r \\ \log_2 m = r \end{array} \quad \left| \quad \begin{array}{l} n = 2^t \\ \log_2 n = t \end{array} \right. \\
 & \log_2 \left(\frac{\sqrt{mn^2}}{16} \right) \\
 &= \log_2 \frac{m^{\frac{1}{2}} n}{16} \\
 &= \frac{1}{2} \log_2 m + \log_2 n - \log_2 2^4 \\
 &= \frac{1}{2} r + t - 4
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{13} \quad \log_8 45 &= \frac{\log_2 45}{\log_2 8} \\
 &= \frac{\log_2 (3^2 \times 5)}{\log_2 2^3} \\
 &= \frac{2 \log_2 3 + \log_2 5}{3 \log_2 2} \\
 &= \frac{2h + k}{3}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{14} \quad \log_3 ab &= \log_3 a + \log_3 b \\
 &= \frac{1}{\log_a 3} + \frac{1}{\log_b 3} \\
 &= \frac{1}{m} + \frac{1}{n} \\
 &= \frac{m + n}{mn}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{15} \quad \log_{21} a &= \frac{1}{\log_a 21} \\
 &= \frac{1}{\log_a (3 \times 7)} \\
 &= \frac{1}{\log_a 3 + \log_a 7} \\
 &= \frac{1}{p + q}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{16} \quad \log_{\sqrt{x}} 9 &= u \\
 \log_9 \sqrt{x} &= \frac{1}{u} \\
 \frac{1}{2} \log_9 x &= \frac{1}{u} \\
 \log_9 x &= \frac{2}{u}
 \end{aligned}$$

$$\log_9 x^3 = 3 \log_9 x = \frac{6}{u}$$

17 $3 = a^p$ $5 = a^q$
 $\log_a 3 = p$ $\log_a 5 = q$

$$\begin{aligned} \log_3 1 \frac{2}{3} &= \log_3 \frac{5}{3} \\ &= \log_3 5 - \log_3 3 \\ &= \frac{\log_a 5}{\log_a 3} - 1 \\ &= \frac{q}{p} - 1 \\ &= \frac{q-p}{p} \end{aligned}$$

18 $\log_{16} 75 = \frac{\log_2 75}{\log_2 16}$
 $= \frac{\log_2 (3 \times 5^2)}{\log_2 2^4}$
 $= \frac{\log_2 3 + 2 \log_2 5}{4}$
 $= \frac{p+2q}{4}$

19 $3^{2x-1} = 4^x$
 $(2x-1) \lg 3 = x \lg 4$
 $2x \lg 3 - x \lg 4 = \lg 3$
 $x(2 \lg 3 - \lg 4) = \lg 3$
 $x = \frac{\lg 3}{2 \lg 3 - \lg 4}$
 $x = 1.355$

20 $3^{\log_3(x+1)} = 5$
 $\log_3(x+1) = \log_3 5$
 $x+1 = 5$
 $x = 4$

21 $7^{\log(x-1)11} = 11$
 $\log_{(x-1)} 11 = \log_7 11$
 $x-1 = 7$
 $x = 8$

22 $\log_5 9 + \log_5 2x - \log_5 (3x+1) = 0$
 $\log_5 \left[\frac{9(2x)}{3x+1} \right] = 0$
 $\frac{9(2x)}{3x+1} = 5^0 = 1$
 $18x = 3x+1$
 $15x = 1$

$$x = \frac{1}{15}$$

23 $\log_{10}(k^2 + 6k + 28) = 2$
 $k^2 + 6k + 28 = 10^2$
 $k^2 + 6k - 72 = 0$
 $(k-6)(k+12) = 0$
 $k = 6$ atau -12

24 $2 + 2 \log_4(p-6) = \log_2 p$
 $2 + 2 \left[\frac{\log_2(p-6)}{\log_2 4} \right] = \log_2 p$
 $2 + 2 \left[\frac{\log_2(p-6)}{2} \right] = \log_2 p$
 $2 + \log_2(p-6) = \log_2 p$
 $\log_2(p-6) - \log_2 p = -2$
 $\log_2 \left(\frac{p-6}{p} \right) = -2$
 $\frac{p-6}{p} = 2^{-2}$
 $\frac{p-6}{p} = \frac{1}{4}$
 $4p - 24 = p$
 $3p = 24$
 $p = 8$

25 $\log_2 m - \log_8 2m = 3$
 $\log_2 m - \frac{\log_2 2m}{\log_2 8} = 3$
 $\log_2 m - \frac{\log_2 2m}{\log_2 2^3} = 3$
 $\log_2 m - \frac{\log_2 2m}{3} = 3$
 $3 \log_2 m - \log_2 2m = 9$
 $\log_2 m^3 - \log_2 2m = 9$
 $\log_2 \left(\frac{m^3}{2m} \right) = 9$
 $\frac{m^2}{2} = 2^9$
 $m^2 = 2^{10}$
 $m = \left(2^{10} \right)^{\frac{1}{2}}$
 $m = 32$

26 $\log_3 2A = \log_9 B + 2$
 $\log_3 2A = \frac{\log_3 B}{\log_3 3^2} + 2$

$$\begin{aligned}\log_3 2A &= \frac{\log_3 B}{2} + 2 \\ 2\log_3 2A &= \log_3 B + 4 \\ 2\log_3 2A - \log_3 B &= 4 \\ \log_3 \frac{(2A)^2}{B} &= 4 \\ \frac{4A^2}{B} &= 3^4 \\ \frac{4A^2}{B} &= 81 \\ A^2 &= \frac{81B}{4} \\ A &= \frac{9\sqrt{B}}{2}\end{aligned}$$

27 $\log_4 x = \log_2 7$

$$\frac{\log_2 x}{\log_2 4} = \log_2 7$$

$$\frac{\log_2 x}{\log_2 2^2} = \log_2 7$$

$$\frac{\log_2 x}{2} = \log_2 7$$

$$\log_2 x = 2\log_2 7$$

$$\log_2 x = \log_2 7^2$$

$$x = 49$$

28 $\log_9 y - \log_3 x = 0$

$$\frac{\log_3 y}{\log_3 9} = \log_3 x$$

$$\frac{\log_3 y}{\log_3 3^2} = \log_3 x$$

$$\frac{\log_3 y}{2} = \log_3 x$$

$$\log_3 y = 2\log_3 x$$

$$\log_3 y = \log_3 x^2$$

$$y = x^2$$

29 $\log_n 1024 - \log_{\sqrt{n}} 2n = 2$

$$\log_n 1024 - \frac{\log_n 2n}{\frac{1}{2}} = 2$$

$$\log_n 1024 - 2\log_n 2n = 2$$

$$\log_n \left(\frac{1024}{4n^2} \right) = 2$$

$$\frac{256}{n^2} = n^2$$

$$n^4 = 256$$

$$n = 4$$

30 (a) $\log_2 150$

$$\begin{aligned}&= \log_2 (2 \times 3 \times 5^2) \\ &= \log_2 2 + \log_2 3 + 2\log_2 5 \\ &= 1 + h + 2k\end{aligned}$$

(b) $\log_4 \left(\frac{125}{9} \right)$

$$\begin{aligned}&= \frac{\log_2 \left(\frac{5^3}{3^2} \right)}{\log_2 4} \\ &= \frac{\log_2 5^3 + \log_2 3^2}{\log_2 2^2} \\ &= \frac{3\log_2 5 + 2\log_2 3}{2} \\ &= \frac{3k - 2h}{2}\end{aligned}$$

31 (a) $\log_5 0.84$

$$\begin{aligned}&= \log_5 \frac{21}{25} \\ &= \log_5 \left(\frac{3 \times 7}{5^2} \right) \\ &= \log_5 3 + \log_5 7 - \log_5 5^2 \\ &= p + q - 2\end{aligned}$$

(b) $\log_7 315 = \frac{\log_5 315}{\log_5 7}$

$$\begin{aligned}&= \frac{\log_5 (3^2 \times 5 \times 7)}{\log_5 7} \\ &= \frac{2\log_5 3 + \log_5 5 + \log_5 7}{\log_5 7} \\ &= \frac{2p + 1 + q}{q}\end{aligned}$$

32 $27^{x+1} = 3^y$

$$\begin{aligned}3^{3(x+1)} &= 3^y \\ 3^{3x+3} &= 3^y\end{aligned}$$

Menyamakan indeks:

$$3x + 3 = y \dots (1)$$

$$\log_3 y = 2 + \log_3 (x-1)$$

$$\log_3 y - \log_3 (x-1) = 2$$

$$\log_3 \left(\frac{y}{x-1} \right) = 2$$

$$\frac{y}{x-1} = 3^2$$

$$y = 9(x-1)$$

$$y = 9x - 9 \dots (2)$$

Gantikan (1) ke dalam (2) :

$$9x - 9 = 3x + 3$$

$$6x = 12$$

$$x = 2$$

Daripada (2) : $y = 9(2) - 9 = 9$

33 $\log_a xy^3 = 9$

$$\log_a x + 3\log_a y = 9$$

$$\log_a x + 3\log_a y = 9$$

Katakan $\log_a x = m$ dan $\log_a y = k$

$$m + 3k = 9 \dots$$

$$m = 9 - 3k \dots (1)$$

$$\log_a x^2 y = 8$$

$$\log_a x^2 + \log_a y = 8$$

$$2\log_a x + \log_a y = 8$$

Katakan $\log_a x = m$ dan $\log_a y = k$

$$2m + k = 8 \dots (2)$$

Gantikan (1) ke dalam (2) :

$$2(9 - 3k) + k = 8$$

$$18 - 6k + k = 8$$

$$18 - 5k = 8$$

$$5k = 18 - 8$$

$$5k = 10$$

$$k = 2$$

Daripada(1) :

$$m = 9 - 3k = 9 - 3(2) = 3$$

$$k = 2$$

$$\log_a y = 2$$

$$m = 3$$

$$\log_a x = 3$$

$$\log_a \sqrt{xy}$$

$$= \log_a (xy)^{\frac{1}{2}}$$

$$= \frac{1}{2}(\log_a x + \log_a y)$$

$$= \frac{1}{2}(3 + 2)$$

$$= \frac{5}{2}$$

34 $3\log_8 (2x+14) - 4\log_{16} (x+1) = 3$

$$3 \left[\frac{\log_2 (2x+14)}{\log_2 8} \right] - 4 \left[\frac{\log_2 (x+1)}{\log_2 16} \right] = 3$$

$$3 \left[\frac{\log_2 (2x+14)}{\log_2 2^3} \right] - 4 \left[\frac{\log_2 (x+1)}{\log_2 2^4} \right] = 3$$

$$\log_2 (2x+14) - \log_2 (x+1) = 3$$

$$\log_2 \left(\frac{2x+14}{x+1} \right) = 3$$

$$\frac{2x+14}{x+1} = 2^3$$

$$2x+14 = 8x+8$$

$$6x = 6$$

$$x = 1$$

35 (a) $2\log_2 (x+y) = 3 + \log_2 x + \log_2 y$

$$\log_2 (x+y)^2 = \log_2 2^3 + \log_2 x + \log_2 y$$

$$\log_2 (x+y)^2 = \log_2 (8xy)$$

$$(x+y)^2 = 8xy$$

$$x^2 + y^2 + 2xy = 8xy$$

$$x^2 + y^2 = 6xy \text{ [Tertunjuk]}$$

(b) $\log_9 [\log_3 (3x-6)] = 5^{\log_5 \left(\frac{1}{2}\right)}$

$$\log_9 [\log_3 (3x-6)] = \frac{1}{2} \leftarrow a^{\log_a x} = x$$

$$\frac{1}{2}$$

$$\log_3 (3x-6) = 9^{\frac{1}{2}}$$

$$\log_3 (3x-6) = 3$$

$$3x-6 = 3^3$$

$$3x-6 = 27$$

$$3x = 33$$

$$x = 11$$

36 $\log \sqrt{x} 9 = a$

$$\log_9 \sqrt{x} = \frac{1}{a}$$

$$\log_9 x^{\frac{1}{2}} = \frac{1}{a}$$

$$\frac{1}{2} \log_9 x = \frac{1}{a}$$

$$\log_9 x = \frac{2}{a}$$

$$\log_y 3 = b$$

$$\frac{\log_9 3}{\log_9 y} = b$$

$$\frac{\frac{1}{2}}{\log_9 y} = b$$

$$\frac{1}{2\log_9 y} = b$$

$$\log_9 y = \frac{1}{2b}$$

$$\begin{aligned} \log_9 xy^2 &= \log_9 x + 2\log_9 y \\ &= \frac{2}{a} + 2\left(\frac{1}{2b}\right) \\ &= \frac{2}{a} + \left(\frac{1}{b}\right) \\ &= \frac{2b+a}{ab} \end{aligned}$$

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Strategi Bijak

Jika 'Sebelah Kanan' lebih rumit daripada 'Sebelah Kiri', adalah lebih baik untuk membuktikan 'Sebelah Kanan' sama dengan 'Sebelah Kiri'.

Sebelah kanan

$$\begin{aligned} &= 2\log_4 x + 2\log_4 y \\ &= \frac{2\log_2 x}{\log_2 4} + \frac{2\log_2 y}{\log_2 4} \\ &= \frac{2\log_2 x}{\log_2 2^2} + \frac{2\log_2 y}{\log_2 2^2} \\ &= \frac{2\log_2 x}{2} + \frac{2\log_2 y}{2} \\ &= \log_2 x + \log_2 y \\ &= \log_2 xy \\ &= \text{Sebelah kiri} \end{aligned}$$

Katakan $\log_4 x = f$ dan $\log_4 y = g$

$$\begin{aligned} \log_2 xy &= 2\log_4 x + 2\log_4 y \\ &= 2f + 2g \end{aligned}$$

$$\log_2 xy = 10$$

$$2f + 2g = 10$$

$$f + g = 5 \dots (1)$$

$$\frac{\log_4 x}{\log_4 y} = \frac{3}{2}$$

$$\frac{f}{g} = \frac{3}{2}$$

$$f = \frac{3}{2}g \dots (2)$$

Gantikan (2) ke dalam (1) :

$$\frac{3}{2}g + g = 5$$

$$\frac{5}{2}g = 5$$

$$g = 2$$

$$\log_4 y = 2$$

$$y = 4^2 = 16$$

Daripada (2) :

$$f = \frac{3}{2}g = \frac{3}{2}(2) = 3$$

$$\log_4 x = 3$$

$$x = 4^3$$

$$x = 64$$

$$38 \log_3 (3x+1) - \log_3 x^2 + \log_9 x^2 = 2$$

$$\log_3 (3x+1) - \log_3 x^2 + \frac{\log_3 x^2}{\log_3 9} = 2$$

$$\log_3 (3x+1) - \log_3 x^2 + \frac{\log_3 x^2}{2} = 2$$

$$2\log_3 (3x+1) - 2\log_3 x^2 + \log_3 x^2 = 4$$

$$\log_3 (3x+1)^2 - \log_3 x^4 + \log_3 x^2 = 4$$

$$\log_3 \frac{(3x+1)^2(x^2)}{x^4} = 4$$

$$\frac{(3x+1)^2(x^2)}{x^4} = 3^4$$

$$(3x+1)^2 = 81x^2$$

$$9x^2 + 6x + 1 = 81x^2$$

$$72x^2 - 6x - 1 = 0$$

$$(6x-1)(12x+1) = 0$$

$$x = \frac{1}{6} \text{ atau } x = -\frac{1}{12}$$

$$x = -\frac{1}{12} \text{ tidak diterima.}$$

$$\therefore x = \frac{1}{6}$$

$$\begin{aligned}
39 \quad & \sqrt{2x+13} - \sqrt{x+10} = 1 \\
& 2x+13 - 2\sqrt{2x+13}\sqrt{x+10} + x+10 = 1 \\
& 3x+22 - 2\sqrt{2x+13}\sqrt{x+10} = 0 \\
& 3x+22 = 2\sqrt{2x+13}\sqrt{x+10} \\
& 9x^2 + 132x + 484 = 4(2x+13)(x+10) \\
& 9x^2 + 132x + 484 = 4(2x^2 + 33x + 130) \\
& 9x^2 + 132x + 484 = 8x^2 + 132x + 520 \\
& x^2 - 36 = 0 \\
& x = 6
\end{aligned}$$

$$\begin{aligned}
40 \quad & MV = P\left(1 + \frac{r}{n}\right)^{nt} = 1\,077\,484 \\
& 800\,000\left(1 + \frac{r}{4}\right)^{4(5)} = 1\,077\,484 \\
& \left(1 + \frac{r}{4}\right)^{20} = 1.346855 \\
& 1 + \frac{r}{4} = 1.346855^{\frac{1}{20}} \\
& 1 + \frac{r}{4} = 1.015 \\
& \frac{r}{4} = 0.015 \\
& r = 0.06, \text{ i.e. } 6\%
\end{aligned}$$

$$\begin{aligned}
41 \quad & 6000(0.95)^n < 3000 \\
& 0.95^n < 0.5 \\
& n \lg 0.95 < \lg 0.5 \\
& n > \frac{\lg 0.5}{\lg 0.95} \\
& n > 13.51 \\
& n_{\text{minimum}} = 14 \text{ tahun}
\end{aligned}$$