Penyelesaian Lengkap

SET 4

Kertas 1

1 (a) $x^2 - (4 - r)x - r^2 = 0$

Katakan punca-punca persamaan kuadratik itu ialah α dan $-\alpha$.

Let the roots of the quadratic equation be α and $-\alpha$.

Hasil tambah punca:

Sum of roots:

$$\begin{aligned} \alpha + (-\alpha) &= 4 - r \\ 0 &= 4 - r \\ r &= 4 \end{aligned}$$

Hasil darab punca

Product of roots

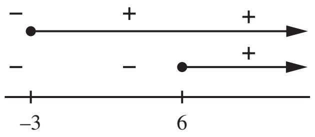
$$\begin{aligned} &= -r^2 \\ &= -4^2 \\ &= -16 \end{aligned}$$

(b) $$\begin{aligned} f(x) &= x^2 - 2hx + 3h + 18 \\ &= (x - h)^2 - h^2 + 3h + 18 \\ &= (x - h)^2 - (h^2 - 3h - 18) \end{aligned}$$

Apabila $f(x) > 0$,

When $f(x) > 0$,

$$\begin{aligned} -(h^2 - 3h - 18) &> 0 \\ h^2 - 3h - 18 &< 0 \\ (h + 3)(h - 6) &< 0 \end{aligned}$$



$-3 < h < 6$

$\therefore a = -3, b = 6$

2 $$\begin{aligned} 3x - 4y + 8z &= 3 \ldots ① \\ x + 2y - 12z &= 17 \ldots ② \\ 7x - 10y + 4z &= 17 \ldots ③ \end{aligned}$$

$② \times 3, \quad 3x + 6y - 36z = 51 \ldots ④$

$④ - ①, \quad 10y - 44z = 48$

$\qquad 5y - 22z = 24 \ldots ⑤$

$② \times 7, \quad 7x + 14y - 84z = 119 \ldots ⑥$

$⑥ - ③, \quad 24y - 88z = 102$

$\qquad 12y - 44z = 51 \ldots ⑦$

$⑦ - ⑤ \times 2, \quad 2y = 3$

$\qquad y = \dfrac{3}{2}$

Daripada ⑤,

From ⑤,

$$\begin{aligned} 5\left(\frac{3}{2}\right) - 22z &= 24 \\ 22z &= -\frac{33}{2} \\ z &= -\frac{3}{4} \end{aligned}$$

Daripada ②,

From ②,

$$\begin{aligned} x + 2\left(\frac{3}{2}\right) - 12\left(-\frac{3}{4}\right) &= 17 \\ x + 3 + 9 &= 17 \\ x &= 5 \end{aligned}$$

$\therefore x = 5, y = \dfrac{3}{2}, z = -\dfrac{3}{4}$

3 $$\begin{aligned} \frac{4(2^{n+1}) - 2^{n+2}}{2^{n+1} - 2^n} &= \frac{4(2^{n+1}) - 2^n(2^2)}{2^{n+1} - 2^n} \\ &= \frac{4(2^{n+1}) - 4(2^n)}{2^{n+1} - 2^n} \\ &= \frac{4(2^{n+1} - 2^n)}{2^{n+1} - 2^n} \\ &= 4 \end{aligned}$$

$$\begin{aligned} \frac{2^{n^2}[4(2^{n+1}) - 2^{n+2}]}{2^{n+1} - 2^n} &= 2^{8n-5} \\ (2^{n^2})(4) &= 2^{8n-5} \\ (2^{n^2})(2^2) &= 2^{8n-5} \\ 2^{n^2+2} &= 2^{8n-5} \\ n^2 + 2 &= 8n - 5 \\ n^2 - 8n + 7 &= 0 \\ (n - 1)(n - 7) &= 0 \end{aligned}$$

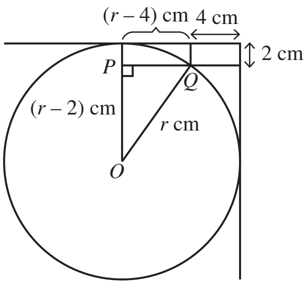
$\therefore n = 1$ atau/*or* $n = 7$

4 (a) $$\begin{aligned} &\left(\frac{9}{3^{\sqrt{6}}}\right)^{2+\sqrt{6}} \\ &= \left(\frac{3^2}{3^{\sqrt{6}}}\right)^{2+\sqrt{6}} \\ &= \left(3^{2-\sqrt{6}}\right)^{2+\sqrt{6}} \\ &= 3^{(2-\sqrt{6})(2+\sqrt{6})} \\ &= 3^{4-6} \\ &= 3^{-2} \\ &= \frac{1}{9} \end{aligned}$$

(b) (i) $$\begin{aligned} xy &= \left(\sqrt{10} + \sqrt{5}\right)\left(2\sqrt{10} - \sqrt{5}\right) \\ &= \sqrt{10}\left(2\sqrt{10}\right) - \sqrt{10}\left(\sqrt{5}\right) + \sqrt{5}\left(2\sqrt{10}\right) - \sqrt{5}\left(\sqrt{5}\right) \\ &= 20 - 5\sqrt{2} + 10\sqrt{2} - 5 \\ &= 15 + 5\sqrt{2} \\ &= 5\left(3 + \sqrt{2}\right) \end{aligned}$$

(ii) $$\begin{aligned} \frac{1}{x} + \frac{1}{y} &= \frac{1}{\sqrt{10} + \sqrt{5}} + \frac{1}{2\sqrt{10} - \sqrt{5}} \\ &= \frac{\left(2\sqrt{10} - \sqrt{5}\right) + \left(\sqrt{10} + \sqrt{5}\right)}{\left(\sqrt{10} + \sqrt{5}\right)\left(2\sqrt{10} - \sqrt{5}\right)} \\ &= \frac{3\sqrt{10}}{5\left(3 + \sqrt{2}\right)} \\ &= \frac{3\sqrt{10}\left(3 - \sqrt{2}\right)}{5\left(3 + \sqrt{2}\right)\left(3 - \sqrt{2}\right)} \\ &= \frac{9\sqrt{10} - 6\sqrt{5}}{5(9 - 2)} \\ &= \frac{9\sqrt{10} - 6\sqrt{5}}{35} \end{aligned}$$

5



$$\begin{aligned}(r-2)^2+(r-4)^2 &= r^2\\ r^2-4r+4+r^2-8r+16 &= r^2\\ r^2-12r+20 &= 0\\ (r-2)(r-10) &= 0\end{aligned}$$

$r = 2$ atau/*or* $r = 10$

$r > 2$, $\therefore r = 10$

Diameter bagi pasu bunga

Diameter of flower vase

$= 2 \times 10$ cm

$= 20$ cm

Diameter bagi pasu bunga adalah lebih besar daripada panjang dan lebar kotak itu.

$\therefore$ Saiz kotak adalah tidak sesuai untuk menyimpan pasu bunga itu.

Diameter of the flower vase is greater than the length and width of the box.

$\therefore$ *The size of the box is not suitable to keep the flower vase.*

6 (a) $y = |a(x+4)(x+1)(x-2)|$

Apabila $x = 0, y = 16,$

When $x = 0, y = 16,$

$$\begin{aligned}16 &= |a(0+4)(0+1)(0-2)|\\ 16 &= |a(4)(1)(-2)|\\ 16 &= |-8a|\\ 8|a| &= 16\\ |a| &= 2\\ a &= \pm 2\end{aligned}$$

$\therefore f(x) = 2(x+4)(x+1)(x-2)$ atau/*or*

$f(x) = -2(x+4)(x+1)(x-2)$

(b) $f(x) = 2x + 3$

Katakan/*Let* $y = 2x + 3$

$$\begin{aligned}2x &= y-3\\ x &= \frac{y-3}{2}\\ \therefore f^{-1}(x) &= \frac{x-3}{2}\end{aligned}$$

$$\begin{aligned}f[g(x)] &= 10x+7\\ g(x) &= f^{-1}(10x+7)\\ &= \frac{(10x+7)-3}{2}\\ &= \frac{10x+4}{2}\\ &= 5x+2\end{aligned}$$

7 (a) $m = -\dfrac{4}{3}$

$y = \dfrac{4}{3}\cos 4\theta$

Apabila $y = 0,$

When $y = 0,$

$$\begin{aligned}\cos 4n &= 0\\ 4n &= \frac{\pi}{2}\\ n &= \frac{\pi}{8}\end{aligned}$$

(b) (i) $\dfrac{1}{2}r^2(2\pi - \theta) = 80$

$$\begin{aligned}2\pi - \theta &= \frac{160}{r^2}\\ \theta &= \left(2\pi - \frac{160}{r^2}\right)\text{ rad}\end{aligned}$$

(ii) Perimeter bagi taman bunga itu

Perimeter of the flower nursery

$$\begin{aligned}&= r(2\pi - \theta) + r + r + 2r\\ &= r\left(\frac{160}{r^2}\right) + 4r\\ &= \left(\frac{160}{r} + 4r\right)\text{ m}\end{aligned}$$

8 (a) $$\begin{aligned}2\underset{\sim}{a} - 3\underset{\sim}{b} &= 2(3\underset{\sim}{i} + \underset{\sim}{j}) - 3(\underset{\sim}{i} + 2\underset{\sim}{j})\\ &= 6\underset{\sim}{i} + 2\underset{\sim}{j} - 3\underset{\sim}{i} - 6\underset{\sim}{j}\\ &= 3\underset{\sim}{i} - 4\underset{\sim}{j}\end{aligned}$$

Vektor unit bagi $2\underset{\sim}{a} - 3\underset{\sim}{b}$

Unit vector of $2\underset{\sim}{a} - 3\underset{\sim}{b}$

$$\begin{aligned}&= \frac{2\underset{\sim}{a} - 3\underset{\sim}{b}}{|2\underset{\sim}{a} - 3\underset{\sim}{b}|}\\ &= \frac{3\underset{\sim}{i} - 4\underset{\sim}{j}}{|3\underset{\sim}{i} - 4\underset{\sim}{j}|}\\ &= \frac{3\underset{\sim}{i} - 4\underset{\sim}{j}}{\sqrt{3^2 + (-4)^2}}\\ &= \frac{3\underset{\sim}{i} - 4\underset{\sim}{j}}{5}\\ &= \frac{3}{5}\underset{\sim}{i} - \frac{4}{5}\underset{\sim}{j}\end{aligned}$$

(b) (i) P, Q dan R ialah titik-titik segaris.

P, Q and R are collinear points.

$$\begin{aligned}\overrightarrow{PQ} &= \lambda\overrightarrow{QR}\\ 4\underset{\sim}{a} + (m-2)\underset{\sim}{b} &= \lambda(6\underset{\sim}{a} - 15\underset{\sim}{b})\\ 4\underset{\sim}{a} + (m-2)\underset{\sim}{b} &= 6\lambda\underset{\sim}{a} - 15\lambda\underset{\sim}{b}\end{aligned}$$

Bandingkan pekali bagi $\underset{\sim}{a}$,

Comparing the coefficients of $\underset{\sim}{a}$,

$4 = 6\lambda$

$\lambda = \dfrac{2}{3}$

Bandingkan pekali bagi $\underset{\sim}{b}$.

Comparing the coefficients of $\underset{\sim}{b}$.

$$\begin{aligned}m-2 &= -15\lambda\\ m-2 &= -15\left(\frac{2}{3}\right)\\ m-2 &= -10\\ m &= -8\end{aligned}$$

(ii) $\overrightarrow{PQ} = \dfrac{2}{3}\overrightarrow{QR}$

$\therefore PQ : QR = 2 : 3$

9 $Y = xy, X = \dfrac{1}{x^2}$

$Y = mX + c$

$c = -7$

$$\begin{aligned}m &= \frac{3-(-7)}{8-0}\\ &= \frac{10}{8}\\ &= \frac{5}{4}\end{aligned}$$

$$\begin{aligned}Y &= \frac{5}{4}X - 7\\ xy &= \frac{5}{4}\left(\frac{1}{x^2}\right) - 7\end{aligned}$$

$$xy = \frac{5-28x^2}{4x^2}$$

$$y = \frac{5-28x^2}{4x^3}$$

Apabila $x = 2.8$,
When $x = 2.8$,

$$y = \frac{5-28(2.8)^2}{4(2.8)^3}$$

$$= -2.44$$

10 (a) $\displaystyle \lim_{x \to 1} \frac{2-2x}{3x^2-5x+2}$

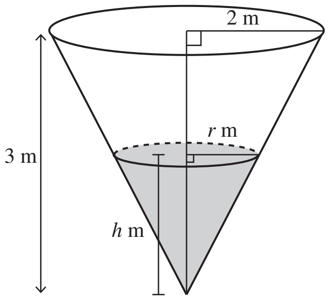
$$= \lim_{x \to 1} \frac{-2(x-1)}{(3x-2)(x-1)}$$

$$= \lim_{x \to 1} \frac{-2}{3x-2}$$

$$= \frac{-2}{3(1)-2}$$

$$= -2$$

(b)



$$\frac{r}{2} = \frac{h}{3}$$

$$r = \frac{2}{3}h$$

$$v = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi\left(\frac{2}{3}h\right)^2 h$$

$$= \frac{4}{27}\pi h^3$$

$$\frac{dv}{dh} = \frac{4}{9}\pi h^2$$

$$\frac{dv}{dt} = \frac{dv}{dh} \times \frac{dh}{dt}$$

$$\frac{dv}{dt} = \frac{4}{9}\pi h^2 \frac{dh}{dt}$$

Apabila $h = 1$, $\dfrac{dv}{dt} = -0.1$,

When $h = 1$, $\dfrac{dv}{dt} = -0.1$,

$$-0.1 = \frac{4}{9}\pi(1)^2 \frac{dh}{dt}$$

$$-\frac{1}{10} = \frac{4}{9}\pi \frac{dh}{dt}$$

$$\frac{dh}{dt} = -\frac{9}{40\pi}$$

$\therefore$ Kadar perubahan bagi tinggi paras air ialah $-\dfrac{9}{40\pi}$ m per minit.

$\therefore$ *The rate of change of the height of water level is* $-\dfrac{9}{40\pi}$ m *per minute.*

11 (a) (i) $\pi(8^2)(4) - \pi\int_2^4 [f(x)]^2\,dx = 200\pi$

$$256\pi - \pi\int_2^4 [f(x)]^2\,dx = 200\pi$$

$$\pi\int_2^4 [f(x)]^2\,dx = 56\pi$$

$$\therefore \int_2^4 [f(x)]^2\,dx = 56$$

(ii) $\pi\int_{-3}^2 [f(x)]^2\,dx = 164\pi - 56\pi$

$$\pi\int_{-3}^2 [f(x)]^2\,dx = 108\pi$$

$$\therefore \int_{-3}^2 [f(x)]^2\,dx = 108$$

(b) $\dfrac{d}{dx}\left(\dfrac{3x}{5-2x}\right) = h(x)$

$$\int h(x)\,dx = \frac{3x}{5-2x} + c$$

$$\int_{-2}^1 [h(x) + 3x^2]\,dx = \int_{-2}^1 h(x)\,dx + \int_{-2}^1 3x^2\,dx$$

$$= \left[\frac{3x}{5-2x}\right]_{-2}^1 + [x^3]_{-2}^1$$

$$= \left[\frac{3(1)}{5-2(1)} - \frac{3(-2)}{5-2(-2)}\right] + [1^3 - (-2)^3]$$

$$= \left[\frac{3}{3} - \frac{-6}{5+4}\right] + [1+8]$$

$$= 1 + \frac{6}{9} + 9$$

$$= 10 + \frac{2}{3}$$

$$= 10\frac{2}{3}$$

12 (a) $T_n = ar^{n-1}$

$$ar^{3-1} = 36$$

$$ar^2 = 36 \ldots ①$$

$$ar^{6-1} = 10\frac{2}{3}$$

$$ar^5 = \frac{32}{3} \ldots ②$$

$$② \div ①: \frac{ar^5}{ar^2} = \frac{\frac{32}{3}}{36}$$

$$r^3 = \frac{8}{27}$$

$$= \left(\frac{2}{3}\right)^3$$

$$r = \frac{2}{3}$$

(b) $\dfrac{S_\infty - S_n}{S_\infty} < 0.0001$

$$\frac{\dfrac{a}{1-r} - \dfrac{a(1-r^n)}{1-r}}{\dfrac{a}{1-r}} < 0.0001$$

$$1 - (1 - r^n) < 0.0001$$

$$r^n < 0.0001$$

$$\left(\frac{2}{3}\right)^n < 0.0001$$

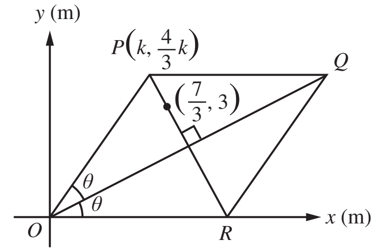
$$n \log\frac{2}{3} < \log 0.0001$$

$$n > \frac{\log 0.0001}{\log\frac{2}{3}}$$

$$n > 22.7$$

$\therefore$ Nilai integer terkecil bagi n ialah 23.
$\therefore$ *The smallest integer value of n is* 23.

13 (a)



Katakan kecerunan bagi $OQ = m$, dengan $m = \tan\theta$

Let gradient of OQ = m, where $m = \tan\theta$

$$\tan 2\theta = \frac{\frac{4}{3}k}{k}$$

$$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{4}{3}$$

$$\frac{2m}{1-m^2} = \frac{4}{3}$$

$$3m = 2 - 2m^2$$

$$2m^2 + 3m - 2 = 0$$

$$(2m-1)(m+2) = 0$$

$m = \frac{1}{2}$ atau/*or* $m = -2$

$m > 0, \therefore m = \frac{1}{2}$

$m_{PR} = -2$

Persamaan bagi pepenjuru PR:

Equation of the diagonal PR:

$$y - 3 = -2\left(x - \frac{7}{2}\right)$$

$$y - 3 = -2x + 7$$

$$y = -2x + 10$$

(b) Gantikan $x = k$, $y = \frac{4}{3}k$ ke dalam $y = -2x + 10$,

Substitute $x = k$, $y = \frac{4}{3}k$ *into* $y = -2x + 10$,

$$\frac{4}{3}k = -2k + 10$$

$$\frac{10}{3}k = 10$$

$$\therefore k = 3$$

(c) Koordinat bagi P ialah $(3, 4)$.

The coordinates of P are $(3, 4)$.

$OP = 5$ m

$PQ = 5$ m

$\therefore$ Koordinat bagi Q ialah $(3 + 5, 4)$ atau $(8, 4)$.

$\therefore$ *The coordinates of Q are* $(3 + 5, 4)$ *or* $(8, 4)$.

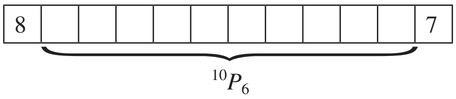
14 (a) Bilangan cara yang berlainan

Number of different ways

$= {}^{12}P_8$

$= 19\,958\,400$

(b)



Bilangan cara yang berlainan

Number of different ways

$= 8 \times {}^{10}P_6 \times 7$

$= 8\,467\,200$

(c) Bilangan cara untuk menyusun dua orang kanak-kanak yang tertentu duduk bersebelahan

Number of ways to arrange two particular children who are to sit next to each other

$= {}^{10}P_7 \times 2!$

$= 1\,209\,600$

Bilangan cara untuk menyusun dua orang kanak-kanak yang tertentu yang tidak boleh duduk bersebelahan

Number of different ways to arrange two particular children who cannot sit next to each other

$= 19\,958\,400 - 1\,209\,600$

$= 18\,748\,800$

15 (a) X = Bilangan bateri motosikal dengan jangka hayat kurang daripada satu tahun

X = *Number of motorcycle batteries with a life span of less than one year*

$X \sim B(4, p)$

$$P(X = x) = {}^4C_x p^x (1-p)^{4-x}$$

$$P(X = 4) = {}^4C_4 p^4 (1-p)^0$$

$$\frac{16}{81} = p^4$$

$$p^4 = \left(\frac{2}{3}\right)^4$$

$$\therefore p = \frac{2}{3}$$

(b) $a = P(X = 0)$

$$= {}^4C_0\left(\frac{2}{3}\right)^0\left(1 - \frac{2}{3}\right)^4$$

$$= \left(\frac{1}{3}\right)^4$$

$$= \frac{1}{81}$$

$b = P(X = 3)$

$$= {}^4C_3\left(\frac{2}{3}\right)^3\left(1 - \frac{2}{3}\right)^1$$

$$= 4\left(\frac{8}{27}\right)\left(\frac{1}{3}\right)$$

$$= \frac{32}{81}$$

(c) $P(1 < X \leqslant 3) = P(X = 2) + P(X = 3)$

$$= \frac{8}{27} + \frac{32}{81}$$

$$= \frac{56}{81}$$

(d) P(bilangan bateri motosikal dengan jangka hayat sekurang-kurangnya satu tahun)

P(*number of motorcycle batteries with a life span of at least one year*)

$$= 1 - \frac{2}{3}$$

$$= \frac{1}{3}$$

Bilangan bateri motosikal dengan jangka hayat sekurang-kurangnya satu tahun

Number of motorcycle batteries with a life span of at least one year

$$= 36 \times \frac{1}{3}$$

$$= 12$$

KERTAS 2

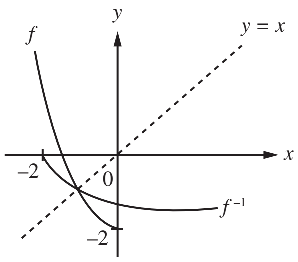
1 (a) $f(x) = x^2 - 2, x \leqslant 0$

$$y = x^2 - 2$$

$$x^2 = y + 2$$

$x \leqslant 0, x = -\sqrt{y + 2}$

$\therefore f^{-1}(x) = -\sqrt{x + 2}, x \geqslant -2$



(b)
$$\begin{aligned} g(x) &= mx+n \\ gf(x) &= 3x^2+11 \\ g(x^2-2) &= 3x^2+11 \\ m(x^2-2)+n &= 3x^2+11 \\ mx^2-2m+n &= 3x^2+11 \end{aligned}$$

Menyamakan pekali bagi x^2,
Equating the coefficients of x^2,
$m = 3$

Menyamakan pemalar,
Equating the constants,
$$\begin{aligned} -2m+n &= 11 \\ -2(3)+n &= 11 \\ -6+n &= 11 \\ n &= 17 \end{aligned}$$

2 (a)
$$\begin{aligned} r\theta &= 8 \ldots ① \\ 15(\theta+0.6) &= 21 \ldots ② \\ \theta+0.6 &= 1.4 \\ \theta &= 0.8 \end{aligned}$$

Daripada ①,
From ①,
$$\begin{aligned} 0.8r &= 8 \\ r &= 10 \end{aligned}$$

(b) Luas bagi rantau berlorek
Area of the shaded region
$$\begin{aligned} &= \frac{1}{2}(15)^2(1.4)-\frac{1}{2}(10)^2(0.8)-\frac{1}{2}(15)(5)\sin 0.6^r \\ &= 157.5-40-21.17 \\ &= 96.33 \text{ cm}^2 \end{aligned}$$

3 (a)
$$\begin{aligned} \overrightarrow{AC} &= \overrightarrow{AD}+\overrightarrow{DC} \\ &= \overrightarrow{AD}+\frac{2}{3}\overrightarrow{AB} \\ &= 4\underset{\sim}{y}+\frac{2}{3}(12\underset{\sim}{x}) \\ &= 8\underset{\sim}{x}+4\underset{\sim}{y} \end{aligned}$$

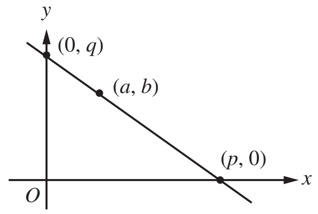
(b) (i)
$$\begin{aligned} \overrightarrow{AF} &= \overrightarrow{AE}+\overrightarrow{EF} \\ &= \frac{1}{3}\overrightarrow{EB}+\lambda\overrightarrow{AD} \\ &= \frac{1}{4}\overrightarrow{AB}+\lambda\overrightarrow{AD} \\ &= \frac{1}{4}(12\underset{\sim}{x})+\lambda(4\underset{\sim}{y}) \\ &= 3\underset{\sim}{x}+4\lambda\underset{\sim}{y} \end{aligned}$$

(ii) Titik-titik *A*, *F* dan *C* adalah segaris.
Points A, F and C are collinear.
$$\begin{aligned} \overrightarrow{AF} &= m\overrightarrow{AC} \\ 3\underset{\sim}{x}+4\lambda\underset{\sim}{y} &= m(8\underset{\sim}{x}+4\underset{\sim}{y}) \\ 3\underset{\sim}{x}+4\lambda\underset{\sim}{y} &= 8m\underset{\sim}{x}+4m\underset{\sim}{y} \end{aligned}$$

Bandingkan pekali bagi $\underset{\sim}{x}$,
Comparing the coefficients of $\underset{\sim}{x}$*,*
$$\begin{aligned} 3 &= 8m \\ m &= \frac{3}{8} \end{aligned}$$

Bandingkan pekali bagi $\underset{\sim}{y}$,
Comparing the coefficients of $\underset{\sim}{y}$*,*
$$\begin{aligned} 4\lambda &= 4m \\ \lambda &= m \\ &= \frac{3}{8} \end{aligned}$$

4 (a)



$$\begin{aligned} m &= \frac{b}{a-p} \\ a-p &= \frac{b}{m} \\ p &= a-\frac{b}{m} \ldots ① \\ m &= -\frac{q}{p} \\ q &= -mp \\ q &= -m\left(a-\frac{b}{m}\right) \ldots ② \\ p+q &= a-\frac{b}{m}-m\left(a-\frac{b}{m}\right) \\ &= a-\frac{b}{m}-am+b \end{aligned}$$

(b) Katakan $r = p+q$
Let $r = p+q$
$$\begin{aligned} r &= a-\frac{b}{m}-am+b \\ \frac{dr}{dm} &= \frac{b}{m^2}-a \\ \frac{d^2r}{dm^2} &= -\frac{2b}{m^3} \end{aligned}$$

Apabila $\frac{dr}{dm} = 0$,

When $\frac{dr}{dm} = 0$,
$$\begin{aligned} \frac{b}{m^2}-a &= 0 \\ \frac{b}{m^2} &= a \\ m^2 &= \frac{b}{a} \end{aligned}$$

$m < 0$, $\therefore m = -\sqrt{\frac{b}{a}}$

Apabila $m = -\sqrt{\frac{b}{a}}$,

When $m = -\sqrt{\frac{b}{a}}$,
$$\begin{aligned} \frac{d^2r}{dm^2} &= -\frac{2b}{m^3} \\ &= -\frac{2b}{\left(-\sqrt{\frac{b}{a}}\right)^3} \\ &= \frac{2(\sqrt{a})^3}{\sqrt{b}} > 0 \end{aligned}$$

$\therefore r = p+q$ adalah minimum apabila $m = -\sqrt{\frac{b}{a}}$.

∴ $r = p+q$ *is minimum when* $m = -\sqrt{\frac{b}{a}}$.

$$r = a + \frac{b}{\sqrt{\frac{b}{a}}} + a\left(\sqrt{\frac{b}{a}}\right) + b$$
$$= a + \sqrt{ab} + \sqrt{ab} + b$$
$$= a + 2\sqrt{ab} + b$$
$$= \left(\sqrt{a}\right)^2 + 2\sqrt{ab} + \left(\sqrt{b}\right)^2$$
$$= \left(\sqrt{a} + \sqrt{b}\right)^2$$

$\therefore$ Nilai minimum bagi $p + q$ ialah $\left(\sqrt{a} + \sqrt{b}\right)^2$.

$\therefore$ *The minimum value of* $p + q$ *is* $\left(\sqrt{a} + \sqrt{b}\right)^2$.

5 (a) (i) n = Bilangan batang buloh yang digunakan

n = Number of bamboo stems that are used

$n \times 5 = 150$

$n = 30$

365, 361, 357, …

Sebutan pertama, $a = 365$ dan beza sepunya $d = -4$.

First term, $a = 365$ and common difference $d = -4$.

$$T_n = a + (n-1)d$$
$$T_{30} = 365 + (30-1)(-4)$$
$$= 365 - 116$$
$$= 249$$

$\therefore$ Panjang bagi batang buloh paling atas yang digunakan ialah 249 cm.

$\therefore$ *The length of the top most bamboo stem that is used is* 249 cm.

(ii) $365 + 361 + 357 + \ldots + 249$

$$S_n = \frac{n}{2}(a + l)$$
$$S_{30} = \frac{30}{2}(365 + 249)$$
$$= 9\,210$$

$\therefore$ Jumlah panjang bagi batang-batang buloh yang digunakan ialah 92.1 m.

$\therefore$ *The total length of the bamboo stems that are used is* 92.1 m.

(b)
$$T_n > 0$$
$$365 + (n-1)(-4) > 0$$
$$365 - 4n + 4 > 0$$
$$4n < 369$$
$$n < 92.25$$

$\therefore n = 92$

Tinggi maximum bagi dinding buloh yang boleh dibina

The maximum height of the bamboo wall that can be constructed

$= 92 \times 5$

$= 460$ cm

6 (a)

Bilangan cara yang berlainan untuk menyusun huruf-huruf V, C, I, N, E, C, O, V, I, D

Number of different ways to arrange the letters V, C, I, N, E, C, O, V, I, D

$= \frac{10!}{2!2!2!}$

$= 453\,600$

$\therefore$ Bilangan cara yang berlainan untuk menyusun huruf-huruf dengan keadaan huruf yang pertama ialah A dan huruf yang akhir ialah C

$\therefore$ *Number of different ways to arrange the letters such that the first letter is* A *and the last letter is* C

$= 1 \times 453\,600 \times 1$

$= 453\,600$

(b) Bilangan cara yang berlainan untuk menyusun huruf-huruf V, A, I, N, E, O, V, I, D, CCC

Number of different ways to arrange the letters V, A, I, N, E, O, V, I, D, CCC

$= \frac{10!}{2!2!}$

$= 907\,200$

Bilangan cara yang berlainan untuk menyusun kesemua huruf C bersebelahan

Number of different ways to arrange all the letters C *next to each other*

$= \frac{3!}{3!}$

$= 1$

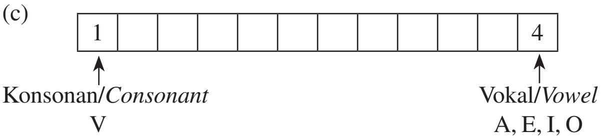
$\therefore$ Bilangan cara yang berlainan untuk menyusun huruf-huruf dengan keadaan semua huruf C adalah bersebelahan

$\therefore$ *Number of different ways to arrange the letters such that all the letters* C *are next to each other*

$= 907\,200 \times 1$

$= 907\,200$

(c)

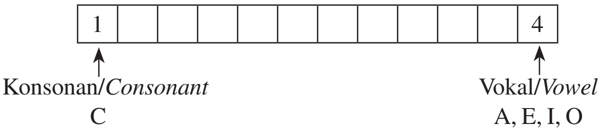


Bilangan cara yang berlainan untuk menyusun huruf-huruf itu dengan huruf yang pertama V dan huruf yang akhir ialah huruf vokal

Number of different ways to arrange the letters with the first letter V *and the last letter is a vowel*

$= 1 \times \frac{10!}{3!2!} \times 4$

$= 1\,209\,600$

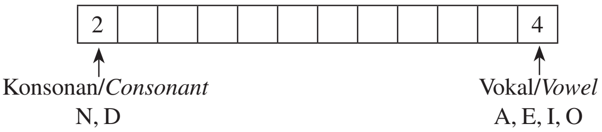


Bilangan cara yang berlainan untuk menyusun huruf-huruf itu dengan huruf yang pertama C dan huruf yang akhir ialah huruf vokal

Number of different ways to arrange the letters with the first letter C *and the last letter is a vowel*

$= 1 \times \frac{10!}{2!2!2!} \times 4$

$= 1\,814\,400$



Bilangan cara yang berlainan untuk menyusun huruf-huruf itu dengan huruf yang pertama N atau D dan huruf yang akhir ialah huruf vokal

Number of different ways to arrange the letters with the first letter N *or* D *and the last letter is a vowel*

$= 2 \times \frac{10!}{2!3!2!} \times 4$

$= 1\,209\,600$

$\therefore$ Bilangan cara yang berlainan untuk menyusun huruf-huruf itu dengan huruf yang pertama ialah huruf konsonan dan huruf yang akhir ialah huruf vokal

$\therefore$ *Number of different ways to arrange the letters such that the first letter is a consonant and the last letter is a vowel*

$= 1\,209\,600 + 1\,814\,400 + 1\,209\,600$

$= 4\,233\,600$

7 (a) X = Bilangan pukulan servis dengan bola mengenai jaring
X = Number of services with the ball hitting the net

$X \sim B(15, k)$

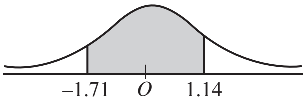
(i)
$$\begin{aligned} \text{Min} &= np \\ \textit{Mean} &= np \\ 4.5 &= 15k \\ k &= 0.3 \end{aligned}$$

(ii) P(sekurang-kurangnya satu bola mengenai jaring)
P(*at least one ball hitting the net*)
$$\begin{aligned} &= P(X \geqslant 1) \\ &= 1 - P(X = 0) \\ &= 1 - {}^{15}C_0(0.3)^0(1 - 0.3)^{15} \\ &= 1 - 0.004748 \\ &= 0.9953 \end{aligned}$$

(b) X = Panjang bagi sejenis baka betik
X = Length of a new species of papaya

$X \sim N(24, 3.5^2)$

$$\begin{aligned} &P(18 < X < 28) \\ &= P\left(\frac{18 - 24}{3.5} < Z < \frac{28 - 24}{3.5}\right) \\ &= P(-1.71 < Z < 1.14) \end{aligned}$$



$$\begin{aligned} &= 1 - P(Z > 1.71) - P(Z > 1.14) \\ &= 1 - 0.0436 - 0.1271 \\ &= 0.8293 \end{aligned}$$

Y = Bilangan betik yang mempunyai saiz biasa dengan panjang antara 18 cm dengan 28 cm
Y = Number of papayas that have normal size with a length between 18 cm *and* 28 cm

$Y \sim B(250, 0.8293)$
$$\begin{aligned} \mu &= np \\ &= 250 \times 0.8293 \\ &= 207.325 \end{aligned}$$

∴ Bilangan betik yang dijangkakan mempunyai saiz biasa ialah lebih kurang 207.
∴ *Expected number of papayas to have normal size is about* 207.

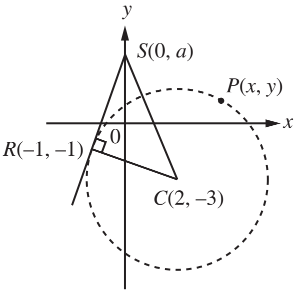
8 (a)
$$\begin{aligned} PC &= RC \\ PC^2 &= RC^2 \\ (x - 2)^2 + (y + 3)^2 &= (-1 - 2)^2 + (-1 + 3)^2 \\ x^2 - 4x + 4 + y^2 + 6y + 9 &= 9 + 4 \end{aligned}$$
$x^2 + y^2 - 4x + 6y = 0$ … ①

∴ Persamaan bagi lintasan titik P ialah $x^2 + y^2 - 4x + 6y = 0$.
∴ *The equation of the path of point P is* $x^2 + y^2 - 4x + 6y = 0$.

(b) Pada titik Q, $x = 5$ dan $y = k$.
Gantikan $x = 5$, $y = k$ ke dalam ①,
At point Q, $x = 5$ and $y = k$.
Substitute $x = 5$, $y = k$ into ①,

$$\begin{aligned} 5^2 + k^2 - 4(5) + 6k &= 0 \\ 25 + k^2 - 20 + 6k &= 0 \\ k^2 + 6k + 5 &= 0 \\ (k + 1)(k + 5) &= 0 \end{aligned}$$
$k = -1$ atau/*or* $k = -5$

(c)



Kecerunan CR
Gradient of CR
$$\begin{aligned} &= \frac{-3 - (-1)}{2 - (-1)} \\ &= \frac{-3 + 1}{2 + 1} \\ &= -\frac{2}{3} \end{aligned}$$

RS dan CR adalah berserenjang.
RS and CR are perpendicular.

∴ Kecerunan $RS = \frac{3}{2}$

∴ *Gradient of RS* $= \frac{3}{2}$
$$\begin{aligned} \frac{a - (-1)}{0 - (-1)} &= \frac{3}{2} \\ a + 1 &= \frac{3}{2} \\ a &= \frac{1}{2} \end{aligned}$$

Luas bagi ΔCRS
Area of ΔCRS
$$\begin{aligned} &= \frac{1}{2}\left|\begin{array}{cccc} 2 & 0 & -1 & 2 \\ -3 & \frac{1}{2} & -1 & -3 \end{array}\right| \\ &= \frac{1}{2}\left|\left[2\left(\frac{1}{2}\right) + 0(-1) + (-1)(-3)\right] - \left[-3(0) + \frac{1}{2}(-1) + (-1)(2)\right]\right| \\ &= \frac{1}{2}\left|[1 - 0 + 3] - \left[0 - \frac{1}{2} - 2\right]\right| \\ &= \frac{1}{2}\left|4 + \frac{5}{2}\right| \\ &= \frac{1}{2}\left|\frac{13}{2}\right| \\ &= \frac{13}{4} \text{ unit}^2/\textit{units}^2 \end{aligned}$$

Kaedah alternatif
Alternative method

$$\begin{aligned} CR &= \sqrt{(2 + 1)^2 + (-3 + 1)^2} \\ &= \sqrt{9 + 4} \\ &= \sqrt{13} \text{ unit/}\textit{units} \end{aligned}$$
$$\begin{aligned} RS &= \sqrt{(-1 - 0)^2 + \left(-1 - \frac{1}{2}\right)^2} \\ &= \sqrt{1 + \frac{9}{4}} \\ &= \sqrt{\frac{13}{4}} \\ &= \frac{\sqrt{13}}{2} \text{ unit/}\textit{units} \end{aligned}$$

Luas bagi ΔCRS
Area of ΔCRS
$$\begin{aligned} &= \frac{1}{2}(\sqrt{13})\left(\frac{\sqrt{13}}{2}\right) \\ &= \frac{13}{4} \text{ unit}^2/\textit{units}^2 \end{aligned}$$

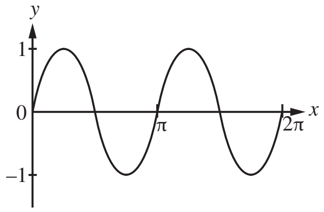
9 (a) $2\cot x\sin^2 x$

$$= 2\left(\frac{\cos x}{\sin x}\right)\sin^2 x$$
$$= 2\cos x\sin x$$
$$= \sin 2x$$

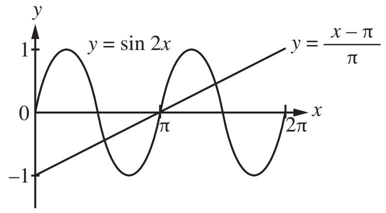
(b)

$$4\cot\left(x-\frac{\pi}{6}\right)\sin^2\left(x-\frac{\pi}{6}\right) = \sqrt{3}$$
$$2\cot\left(x-\frac{\pi}{6}\right)\sin^2\left(x-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$
$$\sin 2\left(x-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$
$$2\left(x-\frac{\pi}{6}\right) = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}$$
$$x-\frac{\pi}{6} = \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$$
$$x = \frac{\pi}{3}, \frac{\pi}{2}, \frac{4\pi}{3}, \frac{3\pi}{2}$$

(c) (i)



$$2\pi\cot x\sin^2 x = x-\pi$$
$$\pi\sin 2x = x-\pi$$
$$\sin 2x = \frac{x-\pi}{\pi}$$
$$y = \frac{x-\pi}{\pi}$$



Garis lurus $y = \frac{x-\pi}{\pi}$ memotong graf $y = \sin 2x$ pada 3 titik.

$\therefore$ Bilangan penyelesaian bagi persamaan $2\pi\,\text{kot}\,x\sin^2 x = x-\pi$ untuk $0 \leqslant x \leqslant 2\pi$ ialah 3.

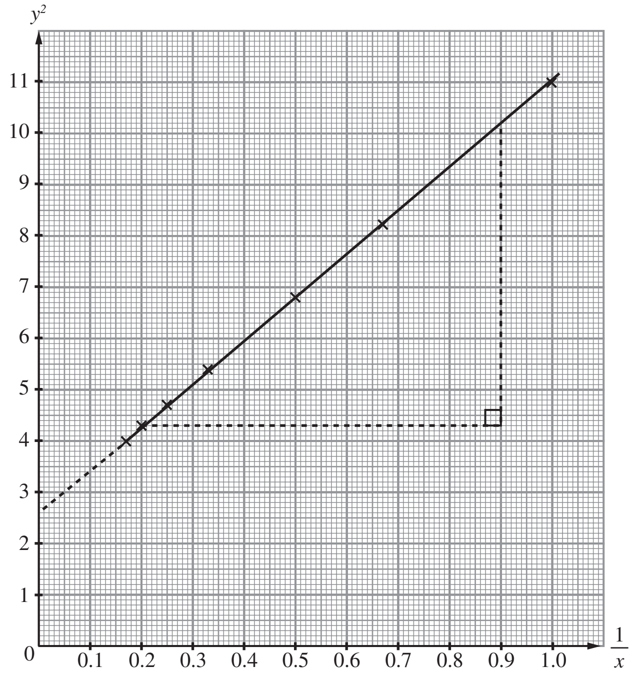
The straight line $y = \frac{x-\pi}{\pi}$ *cuts the graph* $y = \sin 2x$ *at* 3 *points.*

$\therefore$ *The number of solutions for the equation* $2\pi\cot x\sin^2 x = x-\pi$ *for* $0 \leqslant x \leqslant 2\pi$ *is* 3.

10 (a)

$\frac{1}{x}$	1.00	0.67	0.50	0.33	0.25	0.20	0.17
y^2	11.02	8.18	6.81	5.38	4.71	4.28	4.00

(b)



(c) $xy^2 = p + rx$

$$y^2 = p\left(\frac{1}{x}\right) + r$$
$$Y = y^2, X = \frac{1}{x}$$
$$Y = pX + r$$

Pintasan-Y:
Y-intercept:
$r = 2.6$

Kecerunan:
Gradient:

$$p = \frac{10.15-4.3}{0.9-0.2}$$
$$= \frac{5.85}{0.7}$$
$$= 8.36$$

11 (a) $y = 2x - x^2$

$$\frac{dy}{dx} = 2-2x$$

Apabila $\frac{dy}{dx} = 4$,
When $\frac{dy}{dx} = 4$,

$$2-2x = 4$$
$$2x = -2$$
$$x = -1$$

Apabila $x = -1$,
When $x = -1$,

$$y = 2(-1)-(-1)^2$$
$$= -2-1$$
$$= -3$$

$\therefore$ Koordinat bagi titik H ialah $(-1, -3)$.
$\therefore$ *The coordinates of point H are* $(-1, -3)$.

(b) Persamaan tangen pada $H(-1, -3)$:
Equation of the tangent at $H(-1, -3)$:

$$y-(-3) = 4(x+1)$$
$$y+3 = 4x+4$$
$$y = 4x+1$$

Luas bagi rantau berlorek

Area of the shaded region

$= \int_{-1}^{1} [(4x+1)-(2x-x^2)]\,dx$

$= \int_{-1}^{1} (2x+1+x^2)\,dx$

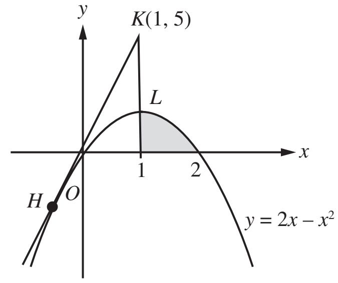
$= \left[x^2 + x + \frac{x^3}{3}\right]_{-1}^{1}$

$= \left[1+1+\frac{1}{3}\right] - \left[1-1-\frac{1}{3}\right]$

$= 2 + \frac{1}{3} + \frac{1}{3}$

$= 2\frac{2}{3}$ unit^2/*units*^{2}

(c)



Isi padu bagi bongkah

Volume of solid

$= \pi\int_{1}^{2} (2x-x^2)^2\,dx$

$= \pi\int_{1}^{2} (4x^2-4x^3+x^4)\,dx$

$= \pi\left[\frac{4x^3}{3} - x^4 + \frac{x^5}{5}\right]_{1}^{2}$

$= \pi\left\{\left[\frac{4(8)}{3} - 16 + \frac{32}{5}\right] - \left[\frac{4}{3} - 1 + \frac{1}{5}\right]\right\}$

$= \pi\left(\frac{28}{3} - 15 + \frac{31}{5}\right)$

$= \frac{8}{15}\pi$ unit^3/*units*^{3}

12 (a) $v = t^3 - 5t^2 + 4t$

$a = \frac{dv}{dt}$

$= 3t^2 - 10t + 4$

Apabila $t = 0$,

When $t = 0$,

$a = 3(0)^2 - 10(0) + 4$

$= 4$

$\therefore$ Pecutan awal bagi zarah itu ialah 4 m s^{-2}.

$\therefore$ *The initial acceleration of the particle is* 4 m s^{-2}.

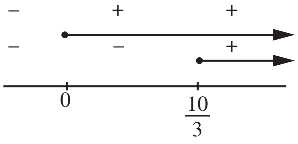
(b) Apabila $a < 4$,

When $a < 4$,

$3t^2 - 10t + 4 < 4$

$3t^2 - 10t < 0$

$t(3t - 10) < 0$



$0 < t < \frac{10}{3}$

$\therefore$ Julat masa apabila pecutan adalah kurang daripada 4 m s^{-2} ialah $0 < t < \frac{10}{3}$.

$\therefore$ *The time interval when the acceleration is less than* 4 m s^{-2} *is* $0 < t < \frac{10}{3}$.

(c) Apabila zarah itu berehat seketika, $v = 0$.

When the particle is instantaneously at rest, $v = 0$.

$t^3 - 5t^2 + 4t = 0$

$t(t^2 - 5t + 4) = 0$

$t(t-1)(t-4) = 0$

$t = 0, t = 1$ atau/*or* $t = 4$

$\therefore$ Zarah itu berehat seketika apabila $t = 0$ s, $t = 1$ s atau $t = 4$ s.

$\therefore$ *The particle is instantaneously at rest when* $t = 0$ s, $t = 1$ s *or* $t = 4$ s.

(d) $s = \int (t^3 - 5t^2 + 4t)\,dt$

$= \frac{t^4}{4} - \frac{5t^3}{3} + 2t^2 + c$

Apabila $t = 0, s = 0, c = 0$

When $t = 0, s = 0, c = 0$

$s = \frac{t^4}{4} - \frac{5t^3}{3} + 2t^2$

Apabila $t = 1$,

When $t = 1$,

$s = \frac{1}{4} - \frac{5}{3} + 2$

$= \frac{7}{12}$ m

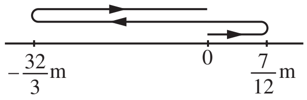
Apabila $t = 4$,

When $t = 4$,

$s = \frac{4^4}{4} - \frac{5(4)^3}{3} + 2(4)^2$

$= 64 - \frac{320}{3} + 32$

$= -\frac{32}{3}$ m



Jumlah jarak yang dilalui

Total distance travelled

$= 2\left(\frac{7}{12}\right) + 2\left(\frac{32}{3}\right)$

$= 22\frac{1}{2}$ m

13 (a) Luas bagi $\Delta ABD = 30$ cm^2

Area of $\Delta ABD = 30$ cm^2

$\frac{1}{2}(7)(10) \sin \angle BAD = 30$

$\sin \angle BAD = \frac{6}{7}$

$\angle BAD = 59°$

(b) $BD^2 = 7^2 + 10^2 - 2(7)(10) \cos 59°$

$= 76.89$

$BD = 8.77$ cm

(c) $\frac{\sin \angle ABD}{10} = \frac{\sin 59°}{8.77}$

$\sin \angle ABD = \frac{10 \sin 59°}{8.77}$

$= 0.9774$

$\angle ABD = 77° 48'$

(d) $\frac{\sin \angle BCD}{8.77} = \frac{\sin 50°}{15}$

$\sin \angle BCD = \frac{8.77 \sin 50°}{15}$

$= 0.4479$

$$\angle BCD = 26^\circ\ 37'$$
$$\angle BDC = 180^\circ - 50^\circ - 26^\circ\ 37' = 103^\circ\ 23'$$

Luas bagi ΔBCD
Area of ΔBCD
$= \frac{1}{2}(8.77)(15) \sin 103^\circ\ 23'$
$= 63.99 \text{ cm}^2$

Luas bagi sisi empat *ABCD*
Area of quadrilateral ABCD
$= 30 + 63.99$
$= 93.99 \text{ cm}^2$

14 (a) $I_{2016/2014} = \frac{P_{2016}}{P_{2014}} \times 100$

$$125 = \frac{x}{3.20} \times 100$$
$$x = 125 \times \frac{3.20}{100} = 4$$

(b) $\frac{w}{y} \times 100 = 160$

$w = 1.6y \ldots$ ①
$w = y + 1.8 \ldots$ ②

$1.6y = y + 1.8$
$0.6y = 1.8$
$y = 3$

Daripada ②,
From ②,
$w = 3 + 1.8$
$= 4.8$

(c) (i) $\bar{I}_{2016/2014} = \frac{P_{2016}}{P_{2014}} \times 100$

$$135 = \frac{67.50}{P_{2014}} \times 100$$
$$P_{2014} = \frac{67.50}{135} \times 100 = 50$$

Harga sekampit baja organik pada tahun 2014 ialah RM50.00.
The price for a beg of organic fertiliser in the year 2014 *was* RM50.00.

(ii)

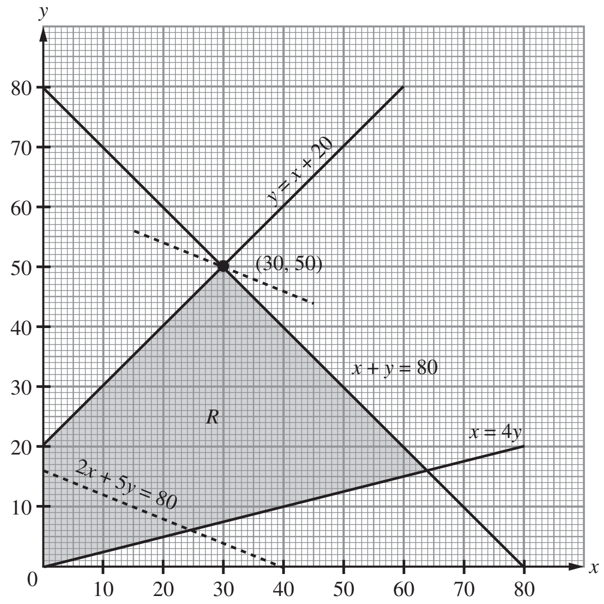
Bahan ***Material***	**Indeks harga pada tahun 2016 berasaskan tahun 2014** ***Price index in the year 2016 based on the year 2014***	**Pemberat** ***Weightage***
P	125	4
Q	$\frac{5.20}{4.00} \times 100 = 130$	*m*
R	160	2
S	$\frac{4.50}{3.00} \times 100 = 150$	1

$$\bar{I} = \frac{\Sigma I_i w_i}{\Sigma w_i}$$
$$135 = \frac{125(4) + 130(m) + 160(2) + 150(1)}{4 + m + 2 + 1}$$
$$135 = \frac{970 + 130m}{m + 7}$$
$$135m + 945 = 970 + 130m$$
$$5m = 25$$
$$m = 5$$

15 (a) x = Bilangan bendera kecil yang digunakan
x = Number of small flags used

y = Bilangan bendera besar yang digunakan
y = Number of big flags used
$x + y \leqslant 80$
$x \leqslant 4y$
$x \geqslant y - 20$
$y \leqslant x + 20$

(b)



(c) (i) Nilai maksimum bagi *x* ialah 64.
∴ Bilangan maksimum bendera kecil yang boleh digunakan ialah 64 keping.
Maximum value of x is 64.
∴ *The maximum number of small flags that can be used is* 64 *pieces.*

(ii) Jumlah kos $= 1.2x + 3y$
Total cost $= 1.2x + 3y$
$1.2x + 3y = 48$
$2x + 5y = 80$

Jumlah kos maksimum
Total maximum cost
$= 1.2(30) + 3(50)$
$=$ RM186